



## TO PREDICT THE EFFECTIVE THERMAL CONDUCTIVITY OF NANOFUIDS FILLED WITH METALLIC NANOPARTICLES CONSIDERING NONLINEAR EFFECT OF VOLUME FRACTION IN SERIES-RESISTOR MODEL

Deepti Chauhan<sup>1</sup>, Seema Bansal<sup>2</sup>, Sumita Shekhawat<sup>3</sup>

<sup>1</sup>Assistant Professor, Department of Physics, Kanoria PG Mahila Mahavidyalaya, Jaipur

<sup>2</sup>Assistant Professor, Department of Physics, JECRC College, Jaipur

<sup>3</sup>Assistant Professor, Department of Physics, Kanoria PG Mahila Mahavidyalaya, Jaipur

---

---

### ABSTRACT

Nanofluids, a mixture of nanoparticles in base fluids, have drawn keen attention in heat transfer applications due to their high thermal conductivity. Pertinent parameters, like fluid and the nanoparticle thermal conductivity, particle size and volume fraction and many others have shown significant but complex and remarkable effects on thermal performance of nanofluids, which is commonly characterized by the thermal conductivity enhancement. We have here, developed a series-resistor model considering various parameters majorly the impact of nanolayer thickness, size, nanoparticle volume fraction and ratio of particle to fluid thermal conductivity. Artificial neural network (ANN) technique has also been used to evaluate the ETC of nanofluids and then the results have been compared with experimental results present in literature and the results evaluated using ANN technique

**Keywords**-Effective thermal conductivity, nanolayer thickness, interfacial resistance, effective volume fraction, artificial neural network technique.

---

---

### [1] INTRODUCTION

Nanofluid is the new, next exciting frontier in technology. The nanofluid technology is still in its early phase and various scientists are working now to use nanofluids as a tool to solve the technological riddles of the modern society. Nanofluids are prepared by dispersing nano-meter sized particles in heat transfer fluid. They have distinctive properties like larger surface to volume ratio, properties that depend on dimension, lower kinetic energy and greater stability. Nanofluids are more stable than micro-milli fluids. Base fluids behave more or less like pure fluids in the presence of nanoparticles thereby incurring very little pressure drop and eliminate the need for surfactants. The most curious property of nanofluid is that they show remarkable enhancement in thermal conductivity even by the

addition of very small amount of nanometre sized particles. The excitement credit goes to the sheer brilliance of the idea and the applications of the technology. Nanofluids have lot of potential applications in microelectronics, fuel cells and pharmaceutical industry. The applications of nanofluids are largely due to the enhanced thermal conductivity. Nanofluids were a result of the experiments intended to increase the thermal conductivity of liquids. The birth of nanofluids is attributed to the revolutionary idea of adding solid particles into heat transfer fluid to increase the thermal conductivity. This innovative idea was first put forth by Maxwell [1] in 1873. Solid particles of micrometre, millimetre magnitudes were added initially to the base fluids to achieve increase in the thermal conductivity but posed a range of serious issues like clogging, increase in the pressure drop, the erosion of pipes and many others. Due to these problems, we couldn't bring about considerable improvement in the practical applications of heat transfer fluids. Numerous theoretical studies have been then conducted [2-8] but failed to predict the thermal conductivity of nanofluid which indicates that some mechanisms have still been missing. Yu and Choi [9] recently brought about remarkable changes by modifying the Maxwell model and introducing a nanolayer concept having thickness of a few nanometres existing at the interface of the particle and the fluid. A formula, then for calculating the effective thermal conductivity (ETC) has been given which was based on effective medium theory and the Maxwell model. This model's predictions have shown that measurable enhancement of the effective thermal conductivity (ETC) can be expected, when nano-layers are accounted for.

In the present paper, a series-resistor model has been developed considering the concept of interfacial nanolayer with linear thermal conductivity distribution. It also includes the effect of nanolayer, nanoparticle size, volume fraction, and thermal conductivity ratio of particle to fluid. The comparisons then have been done between currently available data and predicted values.

## **[2]ARTIFICIAL NEURAL NETWORK (ANN)**

An artificial neural network (ANN) is a data processing system consisting of a large number of highly interconnected processing elements i.e. artificial neurons in an architecture inspired by the structure of the cerebral cortex of the brain. The terminology of artificial neural networks (ANNs) has been developed from a biological model of the brain. A neural network consists of a set of connected cells i.e. the neurons. The neurons get signals from either input cells or from other neurons and then perform some kind of transformation to the input and transmit the outcome to other neurons. The neural networks consist of layers of neurons which are so connected that one layer receives input from the preceding layer of neurons and passes it to the subsequent layer.

Artificial neural network (ANN) is based on feed-forward back-propagation networks with different training functions. Feed-forward back-propagation consists of one or more hidden input layers and an output layer as shown in Fig. 1(a).The signal flows in one direction along connecting pathways in the feed-forward network, from the input layer via hidden layers to the final output layer. The output of each layer is independent of each other so that the output of any layer does not affect that of same or the preceding layer. Back-propagation algorithm (BP) technique is used for training functions. Levenberg-Marquardt (TRAINLM) is used to predict the effective thermal conductivity (ETC) of metal nanoparticles filled in ethylene glycol (EG) and water base-fluids. Back-propagation algorithm (BP) algorithm gives suitable results for subsequent problems. During training the network, calculations were performed from one input network towards output and error was then propagated to prior layers.

**[3]THEORY**

Due to the imperfect contact of the solid-solid interface, the interface resistance becomes a barrier in heat transfer and consequently decrease the overall effective thermal conductivity (ETC). Many studies have already been done on this effect of a solid-solid interface affecting the effective thermal conductivity (ETC) of nanofluid [10-14]. This solid-solid contact resistance phenomenon is not dominant at the solid-liquid interface of particle-in-liquid suspensions. Moreover, it is expected that the nanolayer effect is responsible for the enhancement in thermal conductivity of particle-in-liquid suspensions. To include the effect of the liquid nano-layer, we have here considered a unit cell consisting of a nanoparticle located at the centre surrounded by the base-liquid interacting with the nanoparticles. The nanoparticles are assumed to be mono-dispersed, without agglomeration. Thus the present model considers a nanofluid with low particle concentration, in which inter-particle interactions are neglected. The nanolayer of thickness  $h$  is surrounding the spherical nano particle. The thermal conductivity of the nanolayer is estimated by coupling the heat transfer mechanisms of conduction between nanoparticle and surrounding base-fluid. This solid like nanolayer acts as a thermal bridge between a solid nanoparticle and the fluid. The thermal conductivity of the nanolayer ( $k_{layer}$ ) is assumed to be smaller than the thermal conductivity of nanoparticle but greater than that of base-fluid. Hence the overall system is assumed to consist of a number of small squared elements, each of side  $L$  with a spherical nanoparticle placed at the centre. The heat flow direction is from left into the unit cell as shown in Fig. 1(b). The spherical nanoparticles surrounded by a nanolayer acts as an “equivalent nanoparticle” with a revieve radius  $(r + h)$  and an equivalent thermal conductivity ( $k_{pe}$ ). The equivalent thermal conductivity of equivalent nanoparticle using the effective medium theory [15] which is found to be in good agreement with the experimental data [16] may be given by:

$$k_{pe} = \frac{[2(1-\gamma) + (1+\delta)^3(1+2\gamma)]}{(\gamma-1) + (1+\delta)^3(1+2\gamma)} k_f \tag{1}$$

Where,  $\gamma = k_{layer} / k_p$  is the ratio of nanolayer thermal conductivity to nanoparticle thermal conductivity and  $\delta = h / r$  is the ratio of nanolayer thickness to the original nanoparticle radius.

The increased volume fraction, now called as equivalent nanoparticle volume fraction is given as:

$$\varphi_e = \frac{4\pi(r+h)^3}{3L^3} \tag{2}$$

$$\varphi_e = \varphi(1+\delta)^3 \tag{3}$$

The unit cell is further divided into three parts as part 1 part 2 and part3 shown in the Figure 1(b) having the mean conductivity coefficients  $k_1$ ,  $k_2$  and  $k_3$  respectively.  $k_f$ ,  $k_{pe}$  and  $a_f$ ,

$a_{pe}$  are the conductivity coefficients and the cross-sectional areas with  $q_f$  and  $q_{pe}$  as the heat quantities through the cross-sectional area of base-fluid and the equivalent nanoparticle respectively.

Using the Fourier's law, the heat quantity is expressed as:

$$q = ka \frac{dT}{dx} = \frac{\Delta T}{d/ka} \quad (4)$$

And the thermal resistance (R) is given by:

$$R = d/ka \quad (5)$$

Here d represents the element geometry dimension. For parts one and three

$$k_1 = k_3 = k_f \quad (6)$$

Considering a thin strip of thickness ( $dy$ ) for part two in Figure 1(b) the Fourier law gives:

$$q = q_f + q_{pe} = (k_f a_f + k_{pe} a_{pe}) dT/dy \quad (7)$$

The mean conductivity coefficients for these parts are given below.

As part one and three are identical, conductivity coefficients may be written as:

$$k_1 = k_3 = \int_t k_f dy / t = k_f \quad (8)$$

And for part two, conductivity coefficient may be written as:

$$k_2 = \int_0^{2(r+h)} \left( k_f \frac{a_f}{a} + k_{pe} \frac{a_{pe}}{a} \right) dy / 2(r+h) \quad (9)$$

$$k_2 = k_f \frac{v_f}{2a(r+h)} + k_{pe} \frac{v_{pe}}{2a(r+h)} \quad (10)$$

The various thermal resistances are as under:

$$R_1 = R_3 = \frac{t}{k_f a} \quad (11)$$

And

$$R_2 = \frac{4r^2(1 + \delta)^2}{k_f v_f + k_{pe} v_{pe}} \quad (12)$$

Here  $v_f$  and  $v_{pe}$  are the volumes of base-fluid and equivalent nanoparticle respectively.

Then the total equivalent thermal resistance (R) may be given by:

$$R = R_1 + R_2 + R_3 \quad (13)$$

Substituting the values of resistances from equations (11) and (12) into equation (13) for total equivalent thermal resistance and using the relation between equivalent thermal resistance and conductivity the expression for effective thermal conductivity (ETC) may be written as:

$$k_{eff} = \frac{1}{\frac{1}{k_f} \left\{ 1 - \left( \frac{6\phi_e}{\pi} \right)^{\frac{1}{3}} \right\} + 2 \left\{ k_f \left( \frac{4\pi}{3\phi_e} \right)^{\frac{1}{3}} + \left( \frac{2\phi_e}{9\pi} \right)^{\frac{1}{3}} (k_{pe} - k_f) \right\}^{-1}} \quad (14)$$

**[4] Table 1. –Thermal conductivities of various samples with different sizes**

PolymerMatrix/Filler	Thermal Conductivity of Polymer Matrix/filler (W/m-K)	Size of filler (nm)
Ethylene Glycol(EG)/Iron(Fe)	0.258/80	5
Ethylene Glycol(EG)/ Copper(Cu)	0.258/401	5
Water(H <sub>2</sub> O)/ Copper(Cu)	0.604/401	9
Water(H <sub>2</sub> O)/ Silver(Ag)	0.604/429	10

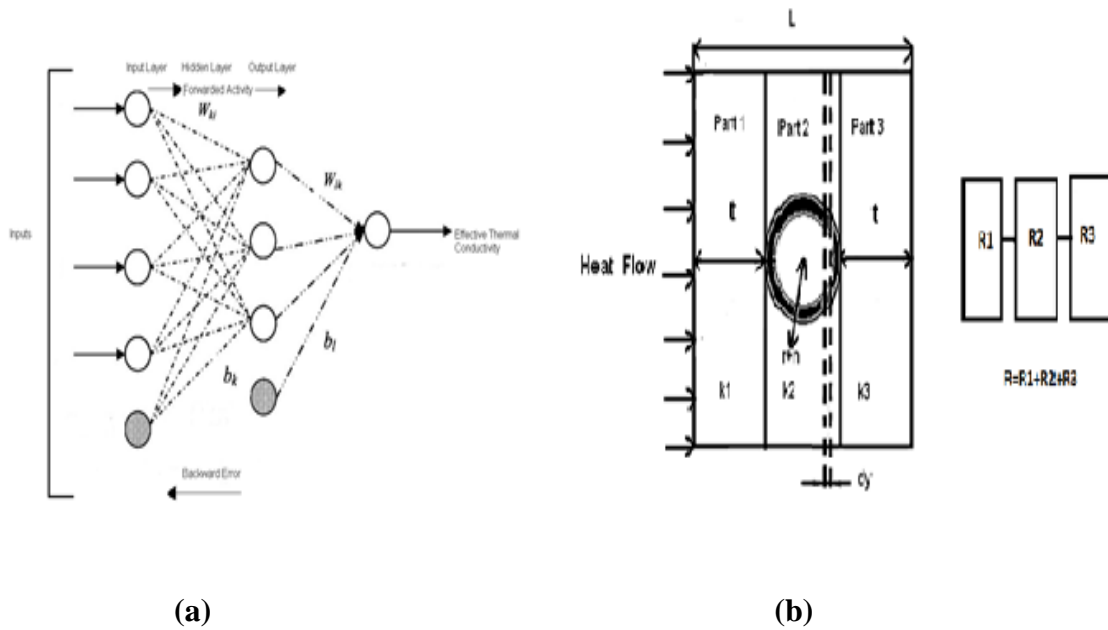
## [5] RESULT AND DISCUSSIONS

Using basic physics of heat transport, a series-resistor model has been developed for which a relevant equation of effective thermal conductivity (ETC) is evaluated. In development of the model, concept of nanolayer at the interface between nanoparticle and base-fluid is considered. The nanolayer has continuous thermal conductivity distribution, that is, it has a

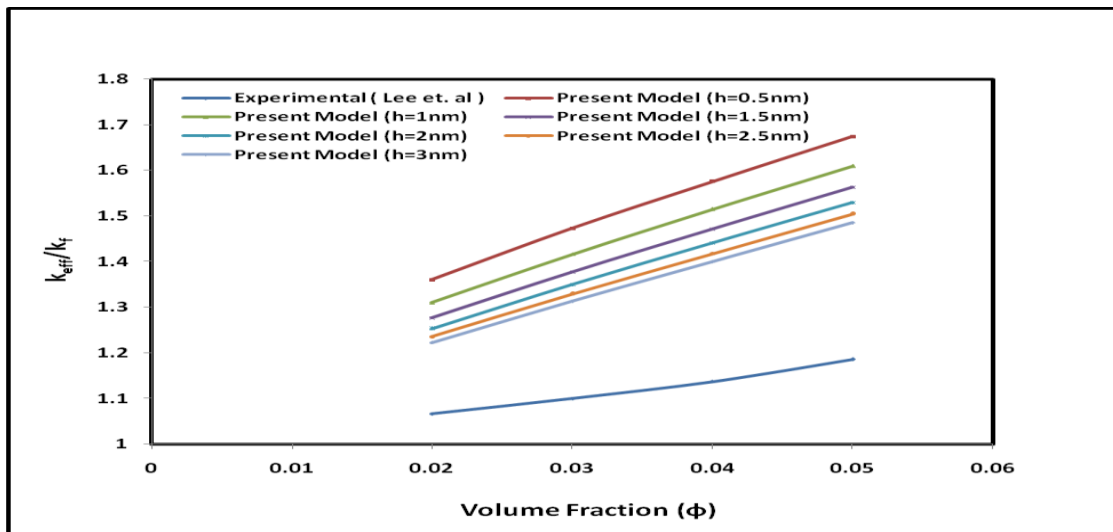
thermal conductivity same to that of nanoparticle at its inner surface. While at the outer surface, its thermal conductivity equates to that of base-fluid. The assumption is based on the concept that the nanolayer is an intermediate physical state between a bulk liquid and a solid nanoparticle [17] and the interaction between the nanoparticle and base-fluid molecules is continuous. Since, there is no available expression for calculating the thermal conductivity of this nanolayer on the surface of the nanoparticle, this solid-like nano layer of base-fluid is expected to have higher thermal conductivity than that of base fluid and therefore it is assumed as  $k_{layer} = 2k_f$ . The interfacial nanolayer thickness  $h$  may be of the order of few atomic diameters [18]. For determining the value of  $h$ , different values of  $h$  is taken for fitting the experimental results and the best fit of  $h$  value is chosen for calculating effective thermal conductivity (ETC) of nanofluids. Figure 2 shows a representative graph for  $Al_2O_3$ -EG ( $r = 19.2$  nm, Lee et al. [19]) from which the interfacial layer thickness is chosen as 2 nm for predicting the effective thermal conductivity (ETC) of nanofluids for metal nanoparticles. The available experimental result for nanofluids filled with metallic nanoparticles is generally for lower volume fraction. Equation (14) well predicts the effective thermal conductivity (ETC) for metal nanoparticles filled nanofluids.

The calculation have been done with water ( $H_2O$ ) and ethylene glycol (EG) base nanofluids filled with metal nanoparticles like silver (Ag), iron (Fe) and copper (Cu), of different sizes. The samples used for calculation purpose are shown in Table 1. Since the filler nanoparticles have higher thermal conductivity than the base fluid, the effective thermal conductivity (ETC) increases rapidly with volume fraction of the filler nanoparticles as shown in Figures (2-6). At lower volume fractions, filler nanoparticles are scattered randomly, but as the volume fraction increases, the filler nanoparticles come in contact with each other and conductive chains are formed in the direction of heat flow. As a result, the effective thermal conductivity (ETC) of nanofluids increases. Further a strong cross attraction between the nanoparticle and base-fluid atoms mediated by the interfacial layer atoms forms a thermal conduction path and hence the effective thermal conductivity (ETC) is increased [23]. It may be seen that the size of the nanoparticles also influences the effective thermal conductivity (ETC) of nanofluid. As heat transfer takes place at the surface of the particle, it is desirable to use particles with larger surface area. The relatively larger surface areas of nanoparticles compared to micro-particles, provide significantly improved heat transfer capabilities. As the size of the filler particles increases, the effective thermal conductivity (ETC) of nanofluids increases which can be seen from the Figures (2-6).

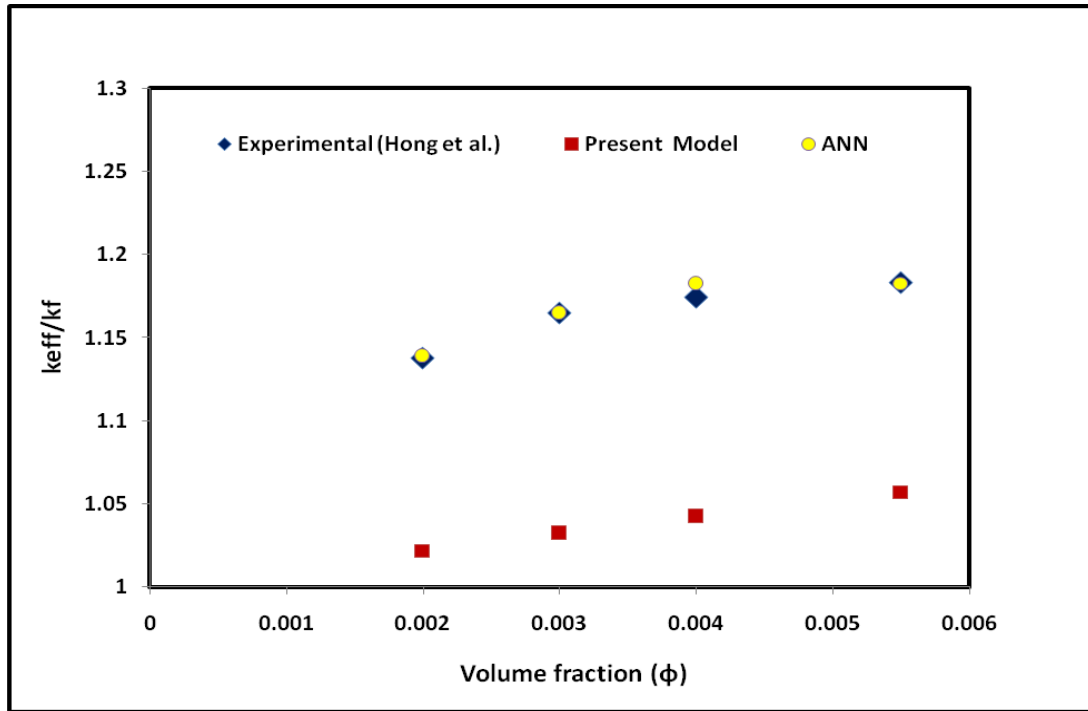
Artificial neural network (ANN) technique has also been used for calculating effective thermal conductivity (ETC) of nanofluids using MATLAB R2010bSP1. A comparison of present model results with the available experimental results and the results by artificial neural network (ANN) technique have been shown in the Figures (3-6) which validates our result. The overall deviation from experimental results is less than 2% to 5% excepting for the sample whose experimental results are shown to be non-linear [20, 23]. There the deviation is within 10%. The deviations from the experimental results may be due to the fact that in the following mechanism, the agglomeration effects are not considered. It can therefore be concluded that the major contributions arise from static mechanisms such as the nanoparticle size, nanoparticle volume fraction, the thickness of the nanolayer and the thermal conductivities of the equivalent nanoparticle and the base fluid and these are shown to play important role in the increment of effective thermal conductivity (ETC) of nanofluids.



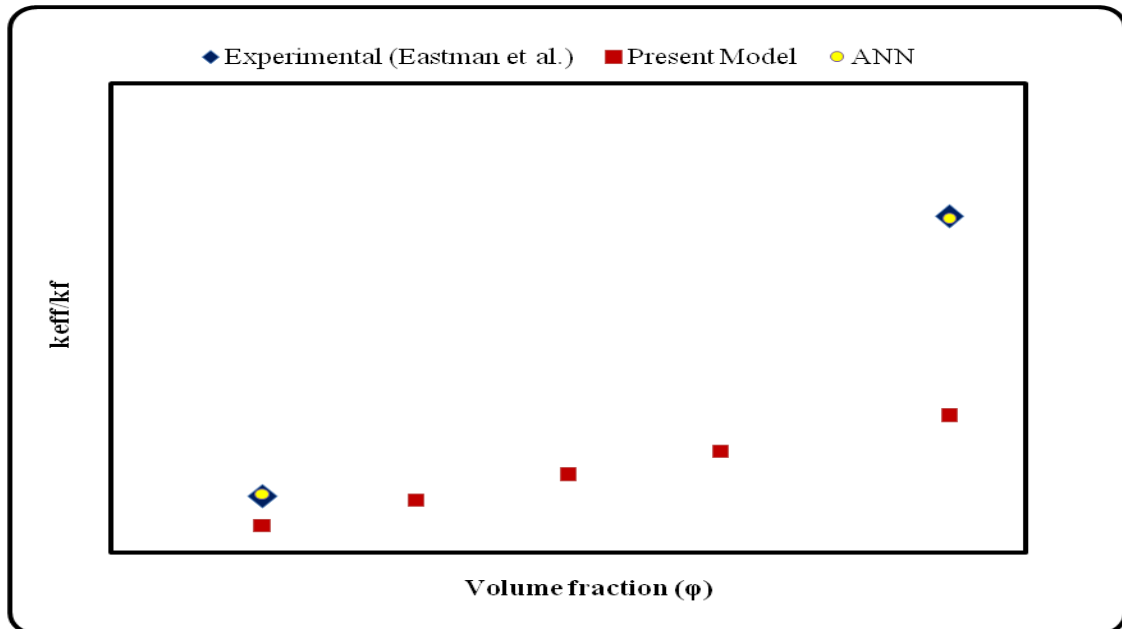
**Fig.-1:** (a) Basic Structure of Artificial Neural Network (ANN) (b) Heat Transfer Series-Resistor Model



**Fig.-2:** Comparison of the present model with experimental results for different values of  $h$  for  $Al_2O_3$ -EG ( $r = 19.2nm$ , Lee et al. [19])

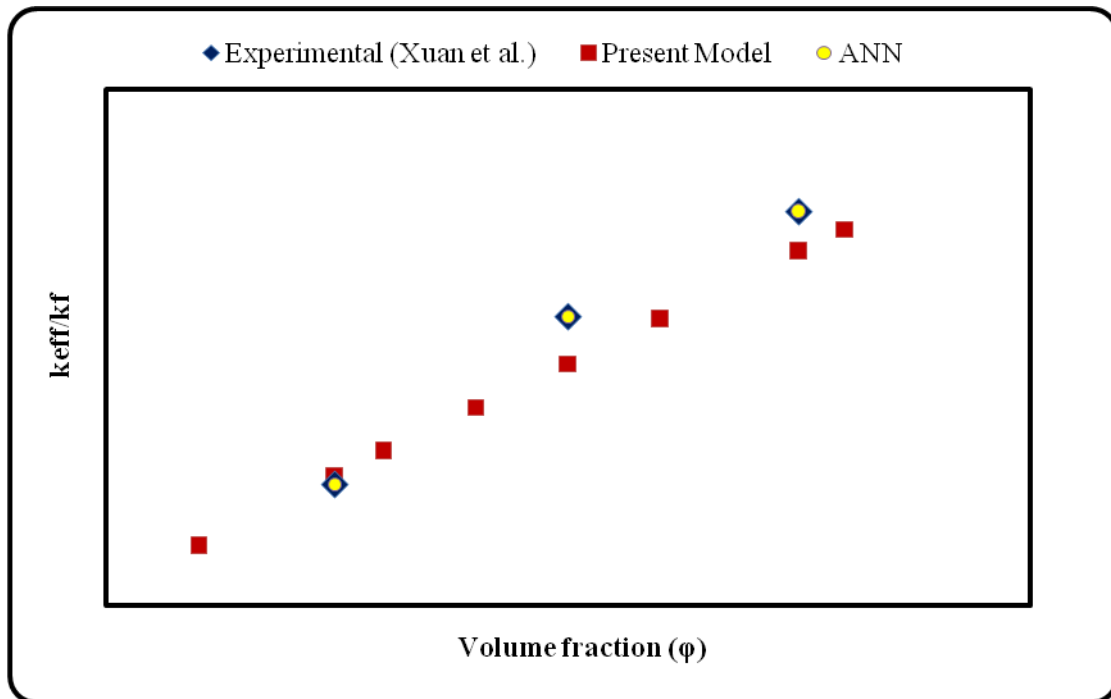


**Fig.-3:** Comparison of present model with the Experimental data and ANN technique: Fe-EG ( $r = 5$  nm Hong et al. [20])

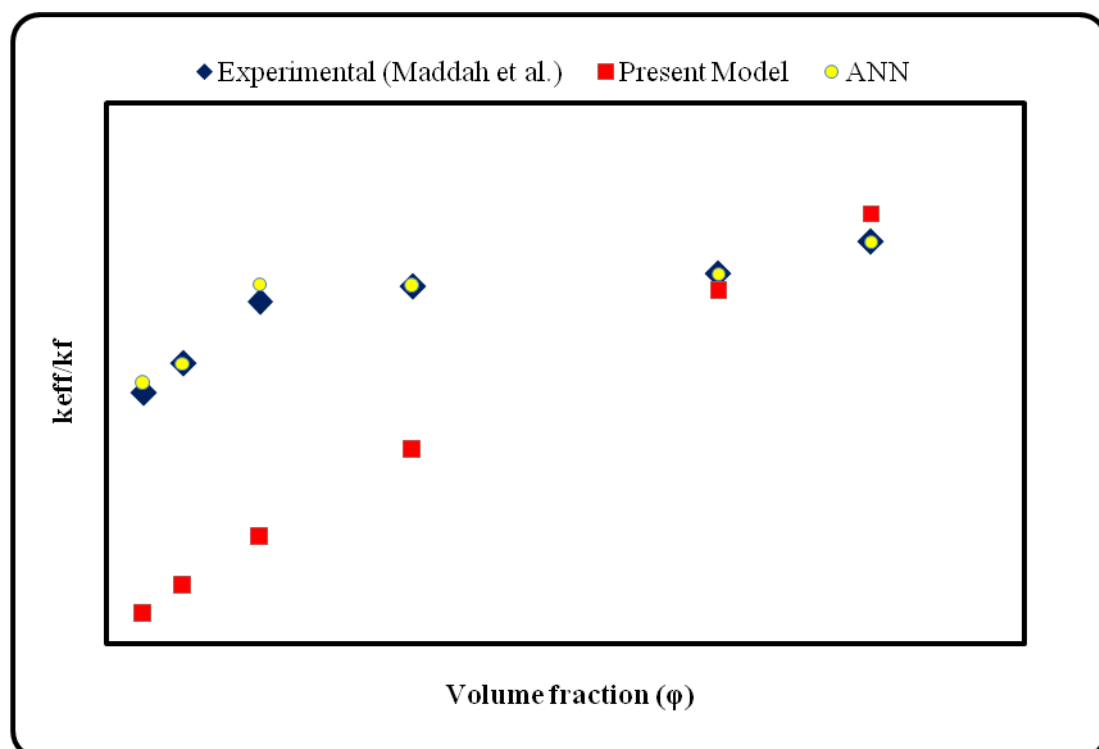


**Fig.-4:** Comparison of present model with the Experimental data and ANN technique: Cu-EG ( $r = 5$  nm Eastman et al. [21])





**Fig.-5:** Comparison of present model with the Experimental data and ANN technique: Cu-Water ( $r = 9$  nm Xuan et al. [22])



**Fig.-6:** Comparison of present model with the Experimental data and ANN technique: Ag-Water ( $r = 10$  nm Maddah et al. [23])

## REFERENCES

- [1] Maxwell J.C.: 'A treatise of Electricity and Magnetism, (Clarendon, Oxford)' Clarendon Press, 1904, 3, pp-404
- [2] Hamilton R.L., Crosser O.K.: 'Thermal Conductivity of Heterogeneous Two Component Systems', *Ind. Eng. Chem. Fundam.*, 1962, 1(3), pp- 187–191
- [3] Davis R.H.: 'The effective thermal conductivity of a composite material with spherical inclusions', *Int. J. Thermophys.*, 1986, 7, pp-609–620
- [4] Hasselman D.H.P., Johnson L.F.: 'Effective Thermal Conductivity of Composites with Interfacial Thermal Barrier Resistance', *J. Compos. Mater.*, 1987, 21(6), pp-508–515.
- [5] Chiew Y.C., Glandt E.D.: 'Effective conductivity of dispersions: The effect of resistance at the particle surfaces', *Chem. Eng. Sci.*, 1987, 42(11), pp- 2677–2685
- [6] Jeffrey D.J.: 'Conduction through a random suspension of spheres', *Proc. Roy. Soc. Lond. A*, 1973, 335(1602), pp- 355–367
- [7] Lu S.Y.: 'Critical interfacial characteristics for effective conductivities of isotropic composites containing spherical inclusions', *J. Appl. Phys.*, 1995, 77(10), 1995, pp-5215–5219
- [8] Lu S.Y., Song J.L.: 'Effective conductivity of composites with spherical inclusions: Effect of coating and detachment', *J. Appl. Phys.*, 1996, 79(2), pp-609–618.
- [9] Yu W., Choi U.S.: 'The Role of Interfacial Layers in the Enhanced Thermal Conductivity of Nanofluids: A Renovated Maxwell Model', *J. Nanoparticle Res.*, 2003, 5, pp-167–171.
- [10] Every A.G., Tzou Y., Hasselman D.P.H., Raj R.: 'The effect of particle size on the thermal conductivity of ZnS/diamond composites', *Acta Metall. Mater.*, 1992, 40(1), pp-123–129.
- [11] Davis L.C., Artz B.E.: 'Thermal conductivity of metal-matrix composites', *J. Appl. Phys.*, 1995, 77(10), pp-4954–4960.
- [12] Torquato S., Rintoul M.D.: 'Effect of the Interface on the Properties of Composite Media', *Phys. Rev. Lett.*, 1995, 75(22-27), pp- 4067–4070.
- [13] Chen G.: 'Size and Interface Effects on Thermal Conductivity of Superlattices and Periodic Thin-Film Structures', *J. Heat Transfer*, 1997, 119(2), pp-220–229.
- [14] Devpura A., Phelan P.E., Prasher R.S.: 'SIZE EFFECTS ON THE THERMAL CONDUCTIVITY OF POLYMERS LADEN WITH HIGHLY CONDUCTIVE FILLER PARTICLES', *Microscale Thermophys. Eng.*, 2001, 5(3), pp-177-189
- [15] Schwartz L.M., Garboczi E.J., Bentz D.P.: 'Interfacial transport in porous media: Application to dc electrical conductivity of mortars', *J. Appl. Phys.*, 1995, 78 (10), pp-5898-5908
- [16] Buongiorno J. et al.: 'A benchmark study on the thermal conductivity of nanofluids', *J. Appl. Phys.*, 2009, 106 (9), pp-094312
- [17] Yu C.J., Richter A.G., Datta A., Durbin M.K., Dutta P.: 'Molecular layering in a liquid on a solid substrate: an X-ray reflectivity study', *Physica B*, 2000, 283(1-3), 27–31
- [18] Leong K.C., Yang C., Murshed S.M.S.: 'A model for the thermal conductivity of nanofluids – the effect of interfacial layer', *J. Nanopart. Res.*, 2006, 8, pp-245-254
- [19] Lee S., Choi S.U.S., Li S., Eastman J.A.: 'Measuring Thermal Conductivity of Fluids Containing Oxide Nanoparticles', *J. Heat Transfer*, 1999, 121 (2), pp-280-289.
- [20] Hong T.K., Yang H.S., Choi C.J.: 'Study of the enhanced thermal conductivity of Fe nanofluids', *J. Appl. Phys.*, 2005, 97 (6), pp-064311 1-4
- [21] Eastman J.A., Choi S.U.S., Li S., Thompson L.J., Lee S.: 'Enhanced Thermal Conductivity through the Development of Nanofluids', *Mater. Res. Soc. Sym. Proc.*, 1997, 457, pp-3-11
- [22] Xuan Y., Li Q.: 'Heat transfer enhancement of nanofluids', *Int. J. Heat Fluid Flow*, 2000, 21(1), pp-158-64
- [23] Maddah H., Rezazadeh M., Maghsoudi, NasiriKokhdan S.: 'The effect of silver and aluminum oxide nanoparticles on thermophysical properties of nanofluids', *J. Nanostructure Chem.*, 2013, 28, pp-1-6