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A STUDY OF FRACTIONAL INTEGRAL OPERATOR AND ITS IMAGE

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ABSTRACT

On account of the importance of the fractional integral operators in the theory of integral equations, in the derivation of functional relations and other allied topics, these operators have been studied from time to time by a number of authors notably Riemann-Liouville and Weyl, Kalla and Saxena [6, 7], Kiryakova [8], Saxena and Kumbhat [11], Kalla [4, 5], Gupta and Soni [1], Jain and Sharma [3] and Jain and Agarwal [2]. A systematic account of fractional integral operators studied by various researchers has been given in the work of Srivastava and Saxena [12].
In the present paper we study a pair of a general class of fractional integral operators involving the product of general sequence of functions, Fox H-function and the $\bar{H}$ -function as their kernels. We first define and give the conditions of existence of the operators of our study and then obtain the images of multivariable H-function under these operators.

Keywords: Fractional Integral operator, Multivariable H-function

1. FRACTIONAL INTEGRAL OPERATORS

$$I_x^{\eta,\rho}\left[f(t)\right] = s x^{-\eta-s\rho-1}\int_0^x t^{\eta}\left(x^s-t^s\right)^{\rho}\,\bar{H}_{P,Q}^{M,N}\left[z\left(\frac{t^s}{x^s}\right)^{\delta_1}\left(1-\frac{t^s}{x^s}\right)^{\delta_2}\right]$$
$$.H_{P',Q'}^{M',N'}\left[z'\left(\frac{t^s}{x^s}\right)^{\sigma_1}\left(1-\frac{t^s}{x^s}\right)^{\sigma_2}\right] R_n^{\alpha,\beta}\left[z''\left(\frac{t^s}{x^s}\right)^{\mu_1}\left(1-\frac{t^s}{x^s}\right)^{\mu_2}\right] f(t)\,dt$$

(1.1)

and

$$J_x^{\eta,\rho}\left[f(t)\right] = s x^{\eta}\int_x^{\infty} t^{-\eta-s\rho-1}\left(t^s - x^s\right)^{\rho} H_{P,Q}^{M,N}\left[z\left(\frac{x^s}{t^s}\right)^{\delta_1}\left(1-\frac{x^s}{t^s}\right)^{\delta_2}\right]$$

$$.H_{P',Q'}^{M',N'}\left[z'\left(\frac{x^s}{t^s}\right)^{\sigma_1}\left(1-\frac{x^s}{t^s}\right)^{\sigma_2}\right] R_n^{\alpha,\beta}\left[z''\left(\frac{x^s}{t^s}\right)^{\mu_1}\left(1-\frac{x^s}{t^s}\right)^{\mu_2}\right] f(t)dt$$

(1.2)

where the function f(t) is such that

$$f(t) = \begin{cases} O\left\{|t|^{\varsigma}\right\}, & |t| \to 0 \\ O\left\{|t|^{w_1} e^{-w_2|t|}\right\}, & |t| \to \infty \end{cases} \qquad (1.3)$$

It has been assumed that operators defined by (1.1) and (1.2) satisfy the following conditions:

(i) s is a positive integer.

(ii) $\min \operatorname{Re}\{\sigma_1, \sigma_2, \mu_1, \mu_2\} \geq 0, \quad \min\{\delta_1, \delta_2, G, h'\} \geq 0$

(iii) $\operatorname{Re}(\rho) + +\mu_2 kn + 1 + \delta_2 \min\limits_{1\leq j\leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) + \sigma_2 \min\limits_{1\leq j\leq M'} \operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right) > 0$

In addition, for the existence of operator (1.1) we need

$$\operatorname{Re}(\eta + \varsigma) + s\,\mu_1 kn + 1 + s\delta_1 \min_{1\leq j\leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) + s\,\sigma_1 \min_{1\leq j\leq M'} \operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right) > 0$$

and for the existence of operator (1.2),

$\operatorname{Re}(w_2) > 0$

or $\operatorname{Re}(w_2) = 0$ and $\operatorname{Re}(\eta - w_1) + s\mu_1 kn + s\delta_1 \min\limits_{1\le j\le M}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right] + s\sigma_1 \min\limits_{1\le j\le M'}\left[\operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right)\right] > 0$

2. FOX H- FUNCTION:

This function will be defined and represented as follows [13]:

$$H_{P',Q'}^{M',N'}[z] = H_{P',Q'}^{M',N'}\left[z\left|\begin{matrix}\left(c'_j,\gamma'_j\right)_{1,P'}\\ \left(d'_j,\delta'_j\right)_{1,Q'}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi\omega}\int_L \phi'(\xi) z^{\xi}\, d\xi \quad , \quad \omega = \sqrt{-1} \qquad (2.1)$$

$$\phi'(\xi) = \frac{\prod_{j=1}^{M'}\Gamma\left(d'_j - \delta'_j\xi\right)\prod_{j=1}^{N'}\Gamma\left(1 - c'_j + \gamma'_j\xi\right)}{\prod_{j=M'+1}^{Q'}\Gamma\left(1 - d'_j + \delta'_j\xi\right)\prod_{j=N'+1}^{P'}\Gamma\left(c'_j - \gamma'_j\xi\right)} \qquad (2.2)$$

The condition of convergence of defining integrals and the other details about the Fox H – function and the $\overline{H}$ - function have been given in the book by Srivastava et al.[13].

3. THE $\overline{H}_{P,Q}^{M,N}$ FUNCTION

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The functions $\overline{H}_{P,Q}^{M,N}$ [.] occurring in (1.1) and (1.2) are defined by the following series representation [9, pp.305-306, Eq.(6.8)]

$$\overline{H}_{P,Q}^{M,N}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N}, (a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M}, (b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right] = \sum_{\nu=1}^{M} \sum_{g=0}^{\infty} \overline{\theta}\left(S_{g,\nu}\right) z^{S_{g,\nu}}$$

(3.1)

where

$$\overline{\theta}\left(S_{g,\nu}\right) = \frac{\prod_{j=1, j\neq\nu}^{M} \Gamma\left(b_j - \beta_j S_{g,\nu}\right) \prod_{j=1}^{N} \left\{\Gamma\left(1-a_j+\alpha_j S_{g,\nu}\right)\right\}^{A_j} (-1)^g}{\prod_{j=M+1}^{Q} \left\{\Gamma\left(1-b_j+\beta_j S_{g,\nu}\right)\right\}^{B_j} \prod_{j=N+1}^{P} \Gamma\left(a_j-\alpha_j S_{g,\nu}\right) g!\, \beta_\nu}, \quad S_{g,\nu} = \frac{b_\nu + g}{\beta_\nu}$$

(3.2)

4. MULTIVARIABLE H-FUNCTION

In the present paper the multivariable H-function will be defined and represented in the following manner [13, p.251]

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$$H\begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{bmatrix} = H\left[z_1,\dots,z_r\right] =$$

$$H^{0,n:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix}\left|\begin{matrix}\left(a'_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)}\right)_{1,p} : \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\dots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(b'_j;\beta_j^{(1)},\dots,\beta_j^{(r)}\right)_{1,q} : \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\dots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{q_r}\end{matrix}\right.\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\dots\int_{L_r}\psi\left(\xi_1,\dots,\xi_r\right)\prod_{i=1}^{r}\left(\phi_i\left(\xi_i\right)z_i^{\xi_i}\right)d\xi_1\dots d\xi_r \tag{4.1}$$

$$\psi\left(\xi_1,\dots,\xi_r\right) = \frac{\prod_{j=1}^{n}\Gamma\left(1-a'_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)}{\prod_{j=1}^{q}\Gamma\left(1-b'_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=n+1}^{p}\Gamma\left(a'_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)} \tag{4.2}$$

$$\phi_i\left(\xi_i\right) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\xi_i\right)\Gamma\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)} \tag{4.3}$$

5. GENARAL SEQUENCE OF FUNCTIONS $R_n^{\alpha,\beta}$ [x]

$R_n^{\alpha,\beta}$ is defined by the following series representation [10]

$$R_n^{\alpha,\beta}\left[x; A, B, C, D; G, h'; \gamma, \delta; \vartheta, k; 1\right] = \sum_{\mu,u,l,e,h} \theta_1(\mu,,u,l,e,h)\, x^{kn+h'(h+\mu)+Gl} \quad (5.1)$$

$$\left|\frac{C}{D} x^{h'}\right| < 1$$

where $\sum_{\mu,,u,l,e,h}$ stands for $\sum_{\mu=0}^{n} \sum_{u=0}^{\mu} \sum_{l=0}^{n} \sum_{e=0}^{l} \sum_{h=0}^{\infty}$ (5.2)

and

$$\theta_1(\mu,u,l,e,h) = A^l B^{\gamma n-l} C^{\mu+h} D^{n\delta-\mu-h} (-l)_e (\alpha)_l (-\mu)_u (-1)^{l+h} (-\alpha-\gamma n)_e (-\beta-n\delta)_\mu$$

$$\cdot \frac{(\mu-\delta n)_h}{k_n (1-\alpha-l)_e \;\; \mu!\, u!\, l!\, e!\, h!} \left(\frac{Ge+\vartheta+h'u}{k}\right)_n kn \quad (5.3)$$

where $\{k_n\}_{n=0}^{\infty}$ is a sequence of constants.

6. MAIN RESULT

IMAGES

In this section, we obtain the images of multivariable H-function in our operators of study:

$$I_x^{\eta,\rho}\left\{ t^{\tau} H^{0,n;m_1,n_1;\ldots;m_r,n_r}_{p,q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 t^{su_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r t^{su_r}\end{matrix}\left|\begin{matrix}\left(a_j^{'};\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \\ \left(b_j^{'};\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right]\right\}$$

$$= x^{\tau}\sum_{\mu,u,l,e,h}\sum_{g=0}^{\infty}\sum_{\nu=0}^{M}\wp\, H^{0,n+2:M',N';m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:P',Q';p_1,q_1;\ldots;p_r,q_r}$$

$$\left[\begin{matrix} z' \\ z_1 x^{su_1} \\ \cdot \\ \cdot \\ z_r x^{su_r}\end{matrix}\left|\begin{matrix} C^{*}:\left(c_j^{'},\gamma_j^{'}\right)_{1,P'};\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \\ D^{*}:\left(d_j^{'},\delta_j^{'}\right)_{1,Q'};\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right]$$

(6.1)

where

$$\wp = \theta_1(\mu,u,l,e,h)x^{\varsigma_1}\,\theta\left(S_{g,\nu}\right)z^{S_{g,\nu}} \qquad (6.2)$$

$$\varsigma_1 = kn + h'(h+\mu) + Gl \qquad (6.3)$$

$$C^{*} = \left(1-\left(\frac{1+\eta+\tau}{s}\right)-\delta_1\left(\frac{b_\nu+g}{\beta_\nu}\right)-\mu_1\varsigma_1;\sigma_1,u_1,\ldots,u_r\right),\left(-\rho-\delta_2\left(\frac{b_\nu+g}{\beta_\nu}\right)-\mu_2\varsigma_1;\sigma_2,0,\ldots,0\right),$$

$$\left(a_j^{'};0,\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p} \qquad (6.4)$$

$$D^{*}=\left(-\rho-\left(\frac{1+\eta+\tau}{s}\right)-\delta_1\left(\frac{b_\nu+g}{\beta_\nu}\right)-(\mu_1+\mu_2)\varsigma_1;\sigma_1+\sigma_2,u_1,\ldots,u_r\right),\left(b_j;0,\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}$$

(6.5)

where $\theta\left(S_{g,\nu}\right)$ is given by (3.2) and the following conditions are satisfied

(i) $\min\operatorname{Re}\{\sigma_1,\sigma_2,\mu_1,\mu_2\}\geq 0,\quad \min\left\{\delta_1,\delta_2,G,h',u_1,\ldots,u_r\right\}\geq 0$

(ii) $\operatorname{Re}(\rho)++\mu_2 kn+1+\delta_2\min\limits_{1\leq j\leq M}\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)+\sigma_2\min\limits_{1\leq j\leq M'}\operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right)>0$

(iii) $\operatorname{Re}(\eta+\varsigma)+s\,\mu_1 kn+1+s\delta_1\min\limits_{1\leq j\leq M}\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)+s\,\sigma_1\min\limits_{1\leq j\leq M'}\operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right)>0$

$$J_x^{\eta,\rho}\left\{t^{\tau}H^{0,n;m_1,n_1;\ldots;m_r,n_r}_{p,q;p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1t^{su_1}\\ \vdots\\ z_rt^{su_r}\end{matrix}\left|\begin{matrix}\left(a'_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \left(b'_j;\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right]\right\}$$

$$=x^{\tau}\sum_{\mu,u,l,e,h}\sum_{g=0}^{\infty}\sum_{\nu=0}^{M}\wp H^{0,n+2:M',N';m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:P',Q';p_1,q_1;\ldots;p_r,q_r}$$

$$\left[\begin{matrix}z_1x^{-su_1}\\ \vdots\\ z_rx^{-su_r}\end{matrix}\left|\begin{matrix}C^{**}:\left(c'_j,\gamma'_j\right)_{1,P'};\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ D^{**}:\left(d'_j,\delta'_j\right)_{1,Q'};\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right]$$

(6.6)

where $\wp$ is given by (6.2), $\bar{\theta}\left(S_{g,\nu}\right)$ is given by (3.2) and C^{**} and D^{**} stand for C^{*}, D^{*} respectively with τ replaced by $-(1+\tau)$ and the following conditions are satisfied

(i) $\min \operatorname{Re}\{\sigma_1, \sigma_2, \mu_1, \mu_2\} \geq 0, \quad \min\{\delta_1, \delta_2, G, h', u_1, \ldots, u_r\} \geq 0$

(ii) $\operatorname{Re}(\rho) + +\mu_2 kn + 1 + \delta_2 \min\limits_{1\leq j\leq M} \left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right] + \sigma_2 \min\limits_{1\leq j\leq M'} \operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right) > 0$

(iii) $\operatorname{Re}(w_2) > 0$ or

$\operatorname{Re}(w_2) = 0$ and

$$\operatorname{Re}(\eta - w_1) + s\mu_1 kn + s\delta_1 \min_{1\leq j\leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) + s\sigma_1 \min_{1\leq j\leq M'} \operatorname{Re}\left(\frac{d'_j}{\delta'_j}\right) > 0$$

PROOF

Using the definition (1.1), the L.H.S. of (6.1) can be written as follows (say Δ)

$$\Delta = sx^{-\eta - s\rho - 1} \int_0^x t^{\eta+\tau} \left(x^s - t^s\right)^{\rho} \bar{H}_{P,Q}^{M,N}\left[z\left(\frac{t^s}{x^s}\right)^{\delta_1}\left(1 - \frac{t^s}{x^s}\right)^{\delta_2}\right].$$

$$.H_{P',Q'}^{M',N'}\left[z'\left(\frac{t^s}{x^s}\right)^{\sigma_1}\left(1 - \frac{t^s}{x^s}\right)^{\sigma_2}\right] R_n^{\alpha,\beta}\left[z''\left(\frac{t^s}{x^s}\right)^{\mu_1}\left(1 - \frac{t^s}{x^s}\right)^{\mu_2}\right] H\left[z_1 t^{su_1}, \ldots, z_r t^{su_r}\right] dt$$

(6.7)

Now, expressing $R_n^{\alpha,\beta}$ and $\overline{H}$-function in their series forms as given by (5.1) and (3.1) respectively, interchanging the order of summation and integration, (6.7) assumes the following form

$$\Delta = \sum_{\mu,u,l,e,h} \sum_{g=0}^{\infty} \sum_{\nu=0}^{M} \wp z^{S_{g,\nu}} s x^{-s\left\{\frac{1+\eta}{s}+\rho+(\delta_1+\delta_2)S_{g,\nu}+(\mu_1+\mu_2)\varsigma_1\right\}} \int_0^x t^{s\left\{\frac{\tau+\eta}{s}+\delta_1 S_{g,\nu}+\mu_1\varsigma_1\right\}}$$

$$.\left(x^s-t^s\right)^{\rho+\delta_2 S_{g,\nu}+\mu_2\varsigma_1} H_{P',Q'}^{M',N'}\left[z'\left(\frac{t^s}{x^s}\right)^{\sigma_1}\left(1-\frac{t^s}{x^s}\right)^{\sigma_2}\right] H\left[z_1 t^{su_1},\ldots,z_r t^{su_r}\right] dt$$

(6.8)

On writing the H-function and the multivariable H-function occurring in the above equation in terms of their Mellin – Barnes contour integrals (2.1) and (4.1), interchanging the orders of contour integrals and t integral, we get

$$\Delta = \sum_{\mu,u,l,e,h} \sum_{g=0}^{\infty} \sum_{\nu=0}^{M} \wp z^{S_{g,\nu}} \frac{1}{(2\pi\omega)^{r+1}} \int_L \int_{L_1} \ldots \int_{L_r} \psi(\xi_1,\ldots,\xi_r) \prod_{i=1}^{r} \left(\phi_i(\xi_i) z_i^{\xi_i}\right) \left(z'\right)^{\xi}$$

$$.sx^{-s\left\{\frac{1+\eta}{s}+\rho+(\delta_1+\delta_2)S_{g,\nu}+(\mu_1+\mu_2)\varsigma_1+(\sigma_1+\sigma_2)\xi\right\}}$$

$$\left\{\int_0^x t^{s\left\{\tau+\eta/s+\delta_1 S_{g,\nu}+\mu_1\varsigma_1+\sigma_1\xi+\sum_{i=1}^{r} u_i\xi_i\right\}}\left(x^s-t^s\right)^{\rho+\delta_2 S_{g,\nu}+\mu_2\varsigma_1+\sigma_2\xi} dt\right\} d\xi\, d\xi_1 \ldots d\xi_r$$

(6.9)

Evaluating the inner t-integral as beta integral and reinterpreting the result thus obtained in terms of multivariable H-function, we arrive at the R.H.S. of (6.1).

The proof of (6.6) can be developed on the similar lines as that of (6.1).

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