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A Novel Method to Design Multirate Filter Banks Using Broyden-Fletcher-Goldfarb-Shanno (BFGS) Optimization

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Abstract-

This paper proposes a new technique for the design of multirate filter banks with linear phase in frequency domain. To match the ideal system response, low-pass analysis prototype filter response is optimized to minimize an objective function. The objective function is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass analysis filter, the square error of the overall transfer function at the quadrature frequency and amplitude distortion of the filter bank. Broyden-Fletcher-Goldfarb-Shanno (BFGS) method is used to minimize the objective function by optimizing the filter tap weights of the prototype filter. Simulation results show that the proposed method is able to perform better than other existing methods.

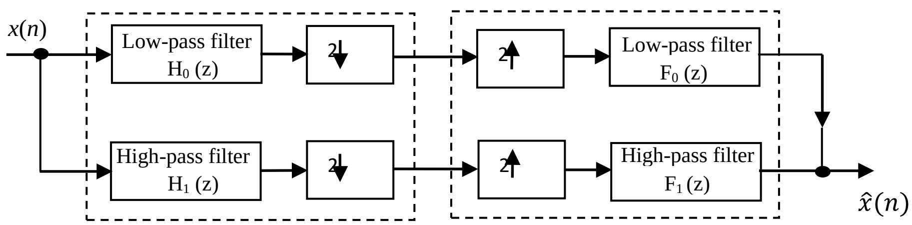
Keywords - Quadrature mirror filter bank, Broyden-Fletcher-Goldfarb-Shanno method, Peak reconstruction error, Linear phase.

1. Introduction

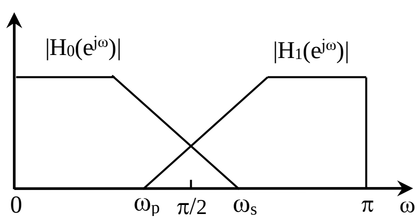
Quadrature mirror filter (QMF) banks are multirate filter bank and have been extensively used for sub-band coding, where the signal is splitinto two or more sub-bands in the frequency domain, so that each sub-band signal can be processed in an independent manner and sufficient compression may be achieved [1]. At the receiver end the sub-band signals are recombined such that the original signal is properly reconstructed [2].QMF banksfind applications in many areas, such as analog to digital conversion [3], design of wavelet bases [4,5], image compression [6,7], digital trans-multiplexers [8], discrete multi-tone modulation systems [9],2-D short-time spectral analysis [10],antenna systems [11], digitalaudio industry [12], biomedical signal processing [13,14,15].

Alias freeefficient design of two-channel QMF banks while keeping minimum dimensions is a tough task.Therefore, various constrained andunconstrained optimization based techniques [16−32] have been developed for the design of linear phase QMF banks.Iterative methods [22−27] and genetic algorithms [28−31] have been proposed for the design problem of QMF based on multi-objective or single objective nonlinear optimization. Authors in [23] have developed efficient technique by considering filter responses in transition band as well as in pass-band and stop-band for the design of QMF bank. Ghosh*et al*. [30] presented an approach based on adaptive-differential-evolution algorithm for the design of two-channel QMF banks.

Fig. 1(a), shows the analysis and synthesis section of a popular multirate filter bank known as two-channel QMF bank. The discrete input signal $x(n)$is dividedinto two sub-band signals having equal band width, using the low-pass and high-pass analysis filters $H_0(z)$and $H_1(z)$, respectively. Typical frequency responses of these filters are depicted in Fig. 1(b). The outputs of the synthesis filters are combined to obtain the reconstructed signal$\hat{x}(n)$. The reconstructed signal $\hat{x}(n)$suffers from three type of errors: aliasing distortion (ALD), phase distortion (PHD), and amplitude distortion (AMD), due to the fact that the filters $H_0(z)$, $H_1(z)$, $F_0(z)$,and$F_1(z)$ are not ideal [32]. Therefore, the main stress of most of the researchers while designing the prototype filter for two-channel QMF bank has been on the elimination or minimization of these three distortions to obtain a perfect reconstruction (PR) or nearly perfect reconstruction (NPR) system [2, 16-21].
The overall transfer function of such an alias and phase distortion free system turns out to be a function of the filter tap coefficients of the low-pass analysis filter only [23]. Then, the AMD can only be minimized by optimizing the filter tap weights of the low-pass analysis filter using computer assistance techniques [2].



(a)



(b)

Fig.1 (a) Two-channel quadrature mirror filter bank
(b) Typical frequency responses of the analysis filters $H_0(z)$ and $H_1(z)$.

It is well known [23] that the relation between output and input of analias free two-channel QMF bank. According [23], the condition for perfect reconstruction can be written as

$|T(\omega)| = |H_r(\omega)|^2 + |H_r(\pi - \omega)|^2 = c$,for all ω.(1)

If the amplitude response of QMF bank in transition band is optimized, then the overall performance can be improved. Therefore, the aim is to optimize the coefficients of $H_0(z)$ by systematic computer-aided optimization technique, such that the filters satisfy the perfect reconstruction condition of Eq.(1) approximately.

In the above context, this paper presents a novel method todesign amultirate filter bank. The objective function to be minimized [23] is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass prototype filter, the square error of the overall transfer function at the quadrature frequency ω = $\pi/2$ and amplitude distortion of the QMF bank. Broyden-Fletcher-Goldfarb-Shanno (BFGS) method is used to minimize the objective function by optimizing the filter tap weights of the prototype filter. The organization of rest of paper isas follows.Section 2 describes formulation of the design problem. Section 3presents proposed algorithm for design of prototype filter. Section 4 discusses the design results of the filter bank and comparison with already existing methods. Finally, conclusions are drawn in section 5.

2. Design Problem Formulation

To satisfy the perfect reconstruction condition of Eq. (1), an overall error function 'E' to be minimized is formulated as a weighted sum of four terms shown below:

$$E = \alpha_1 . E_p + \alpha_2 . E_s + \alpha_3 . E_t + \alpha_4 . E_{am} \quad (2)$$

whereα_1, α_2 ,α_3 and α_4 are real constants, and E_p, E_s,$E_t$and $E_{am}$are the measure of pass-band error, stop-band residual energy, square error of the overall transfer function at $\pi/2$, in transition band, andamplitude distortion, respectively. The objective function $E$is to be minimized iteratively using BFGS method by optimizing the coefficients of prototype filter.
For perfect reconstruction, the overall amplitude response of QMF bank,for all ω,must be equal to square of amplitude response of prototype filter at zero frequency. Consequently, the PR condition of Eq. (1) at quadrature frequency $\omega = \pi/2$, is reduced to

$$H_r\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} H_r(0) \quad (3)$$

where$H_r(0)$ and $H_r(\pi/2)$ are the amplitude response of prototype filter at zero frequency and quadrature frequency, respectively.The square error $E_t$is given by

$$E_t = [H_r\left(\frac{\pi}{2}\right) - \frac{1}{\sqrt{2}} H_r(0)]^2 \quad (4)$$

Further, E_p,E_s and E_{am} are defined as follows:

$$E_p = \frac{1}{\pi} \int_0^{\omega_p} \left[|H_r(0)| - |H_r(\omega)| \right]^2 d\omega \quad (5)$$

$$E_s = \frac{1}{\pi} \int_{\omega_s}^{\pi} \left| H_0(e^{j\omega}) \right|^2 d\omega \quad (6)$$

$$E_{am} = \max_{\omega} |T(e^{j\omega})| - \min_{\omega} |T(e^{j\omega})| \quad (7)$$

For even filter length (N+1), the frequency response of low-pass prototype filter$H_0(e^{j\omega})$ is given by [2]

$$H_0(e^{j\omega}) = \left[\sum_{n=0}^{(N-1)/2} 2h_0(n) \cos\omega\left((N/2) - n\right) \right] . e^{-j\omega N/2} \quad (8)$$

$= H_r(\omega)\, e^{-j\omega N/2}$ (9)

where $b(n) = 2h_0(n)$ and $H_r(\omega)$ is the amplitude function written as

$H_r(\omega) = \mathbf{b}^{\mathbf{T}}\mathbf{c}(\boldsymbol{\omega})$(10)

where vectors **b** and **c(ω)** are:

$\boldsymbol{b} = [b_0\, b_1\, b_2\ \ldots \ldots b_{(N-1)/2}]^T,\quad \mathbf{c}(\boldsymbol{\omega}) = [cos\ \omega(N/2)\ cos\omega((N/2)-1) \ldots \ldots \ldots\ cos\ (\omega/2)]^T$(11)

The amplitude function at zero frequency $\omega = 0$ is calculated as

$H_r(0) = \mathbf{b}^{\mathbf{T}}\mathbf{c}\ (\mathbf{0}) = \mathbf{b}^{\mathbf{T}}\mathbf{1},$ (12)

where **1** is the vector with all $(N+1)/2$ elements equal to unity.

Now, E_t, E_p and E_s can be realized as:

$E_t = [\mathbf{b}^{\mathbf{T}}\mathbf{c}(\boldsymbol{\pi}/\mathbf{2}) - \frac{1}{\sqrt{2}}\mathbf{b}^{\mathbf{T}}\mathbf{c}(\mathbf{0})]^2 \quad = \quad [\mathbf{b}^{\mathbf{T}}\mathbf{d} - H_{r1}]^2$ (13)

where vector **d** and **c(0)** are equal to vector **c(ω),** when it is evaluated at $\omega = \pi/2$ and $\omega = 0$, respectively, and $H_{r1} = 0.707\ \mathbf{b}^{\mathbf{T}}\mathbf{c}(\mathbf{0})$.

Similarly E_p can be realized

$E_p = \mathbf{b}^{\mathbf{T}}\mathbf{F}\ \mathbf{b}$(14)

where **F** is a real, symmetric and positive definite matrix, given by

$$\mathbf{F} = \frac{1}{\pi}\int_0^{\omega_p} (\mathbf{c}(\mathbf{0}) - \mathbf{c}(\boldsymbol{\omega}))(\mathbf{c}(\mathbf{0}) - \mathbf{c}(\boldsymbol{\omega}))^{\mathbf{T}} d\omega \quad (15)$$

Stop band error E_s is given as:

$$E_s = \mathbf{b}^{\mathbf{T}}\ \mathbf{G}\ \mathbf{b} \quad (16)$$

where **G** is a real, symmetric and positive definite matrix, calculated as

$$\mathbf{G} = \frac{1}{\pi}\int_{\omega_s}^{\pi} \mathbf{c}(\boldsymbol{\omega})\mathbf{c}(\boldsymbol{\omega})^{\mathbf{T}} d\omega \quad (17)$$

3. Proposed Method for Design of Low- PassPrototype Filter

By using Eqs.(13), (14) and (16) the objective E can be rewritten as

$E = \alpha_1\mathbf{b}^{\mathbf{T}}\mathbf{F}\ \mathbf{b} + \alpha_2\mathbf{b}^{\mathbf{T}}\ \mathbf{G}\ \mathbf{b} + \alpha_3[\mathbf{b}^{\mathbf{T}}\mathbf{d} - H_{r1}]^2 + \alpha_4 E_{\boldsymbol{am}}$(18)

$= \alpha_1\mathbf{b}^{\mathbf{T}}\mathbf{F}\ \mathbf{b} + \alpha_2\mathbf{b}^{\mathbf{T}}\ \mathbf{G}\ \mathbf{b} + \alpha_3[\mathbf{b}^{\mathbf{T}}\mathbf{D}\mathbf{b} - 2H_{r1}\mathbf{b}^{\mathbf{T}}\mathbf{d} + H_{r1}{}^{\mathbf{2}}] + \alpha_4 E_{\boldsymbol{am}}$

$= \mathbf{b}^{\mathbf{T}}\mathbf{R}\ \mathbf{b} + \alpha_3[-2H_{r1}\mathbf{b}^{\mathbf{T}}\mathbf{d} + H_{r1}{}^{\mathbf{2}}] + \alpha_4 E_{\boldsymbol{am}}$(19)

where matrix **R**and **D**are

$\mathbf{R} = \alpha_1\mathbf{F} + \alpha_2\,\mathbf{G} + \alpha_3\mathbf{D}$and $\quad \mathbf{D} = \mathbf{d}\mathbf{d}^{\mathrm{T}}$

The objective function given by Eq. (18) is a quadratic function and matrix**R** is a Hermitianpositive definite matrix, therefore,E can be minimized by BFGS method [33]. If $\mathbf{b}_i$ is the approximation of the minimum point at the ith stage of iteration and λ_i is the optimal step length in the search direction, then the new or improved approximation in the (i+1)th stage of iteration using BFGS method can be calculated as

$$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i[\mathbf{B}_i]\nabla E_i \;=\; \mathbf{b}_i + \lambda_i\mathbf{s}_i \qquad (20)$$

where∇E_i is the gradient of the objective function E and $\mathbf{s}_i$is the search direction, when evaluated at the design vector $\mathbf{b}_i$,both are given by

$$\nabla E_i \;=\; 2\mathbf{Rb} + \alpha_3[-2H_{r1}\mathbf{d}]\text{and}\,\mathbf{s}_i = -[\mathbf{B}_i]\nabla E_i \quad (21)$$

and matrix [$\mathbf{B}_i$] is the estimate of inverse of Hessian matrix. Initially the matrix [$\mathbf{B}_i$] is taken as the identity matrix [**I**] and up-dation of this matrix is done using BFGS formula [33].

$$[\mathbf{B}_{i+1}] = [\mathbf{B}_i] + \left[1 + \frac{\mathbf{g}_i^T[\mathbf{B}_i]\mathbf{g}_i}{\mathbf{d}_i^T\mathbf{g}_i}\right]\frac{\mathbf{d}_i\mathbf{d}_i^T}{\mathbf{d}_i^T\mathbf{g}_i} - \frac{\mathbf{d}_i\mathbf{g}_i^T[\mathbf{B}_i]}{\mathbf{d}_i^T\mathbf{g}_i} - \frac{[\mathbf{B}_i]\mathbf{g}_i\mathbf{d}_i^T}{\mathbf{d}_i^T\mathbf{g}_i} \quad (22)$$

where

$$\mathbf{d}_i = \mathbf{b}_{i+1} - \mathbf{b}_i = -\lambda_i[\mathbf{B}_i]\,\nabla E_i\text{and}\,\mathbf{g}_i = \nabla E_{i+1} - \nabla E_i \quad (23)$$

The optimum step length λ_i in the direction of $\mathbf{s}_i$ can be obtained by equating the derivate of objective function $E(\mathbf{b}_i + \lambda_i\mathbf{S}_i)$ with respect to λ, to zero. The derivate $dE/d\lambda = 0$, gives following expression for optimum step length

$$\lambda_i = \{\mathbf{b}_i^T\mathbf{R}\mathbf{s}_i - \alpha_3 H_{r1}\mathbf{d}^{\mathrm{T}}\mathbf{s}_i\}\big/\{\mathbf{s}_i^T\mathbf{R}\mathbf{s}_i\} \quad (24)$$

The unit energy constraint on the filter coefficients is also imposed within some prespecified limit as proposed in [17,27].The design algorithm that minimizes the objective function in step by step manner proceeds through following steps:

(1) Specify filter length (N+1), stop band edge frequency (ω_s) and pass band edge frequency (ω_p).
(2) Assume initial values of $\alpha_1, \alpha_2, \alpha_3$ and α_4.
(3) Start with an initial design vector $\mathbf{h}_0$ = [0 0 0 0 0 …………$\frac{1}{\sqrt{2}}$], $\mathbf{h}_0$ is zero except $\mathbf{h}_0((N-1)/2) = \frac{1}{\sqrt{2}}$; satisfying the unit energy constraint within a pre-specified tolerance given by

$$g_1 = \left|1 - 2\sum_{k=0}^{(N-1)/2} h_0^2(k)\right| \le \varepsilon_1$$

(4) Set the iteration number, i = 1, and $\mathbf{b}_i = 2\mathbf{h}_0$.
(5) Compute the objective function E_i, by using Eq. (18), at the design vector $\mathbf{b}_i$.
(6) Compute $\nabla E_i$and the search direction $\mathbf{s}_i$,, at the design vector $\mathbf{b}_i$.
(7) Determine the optimum step length λ_i, by using Eq. (24).
(8) Compute the new approximation

$$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i[\mathbf{B}_i]\nabla E_i \;=\; \mathbf{b}_i + \lambda_i\mathbf{s}_i$$

(9) Obtain the constraint g_1, at the point $\mathbf{b}_{i+1}$, if it is violated then choose the optimum point as $\mathbf{b}_i$, stop the algorithm and go to step (13).

(10) Compute the amplitude distortion E_{am} using Eq. (13) at the design vector $\mathbf{b_{i+1}}$.
(11) Compute the objective function E_{i+1} and ∇E_{i+1}at the design vector $\mathbf{b_{i+1}}$. Also compute matrix [B_{i+1}]. If $E_{i+1} \geq E_{i}$,,choose the optimum point as $\mathbf{b_i}$, stop the procedure and go to step (13). If $E_{i+1} < E_{i}$,set $E_i= E_{i+1}$,and$\mathbf{b_i}= \mathbf{b_{i+1}}$.
(12) Set the new iteration number as $i = i + 1$, and go to step (5).
(13) The optimum solution is$\mathbf{h_0 = (1/2) b_i}$, and stop the procedure.

4. Results and Discussion

Design example is presentedin this section to illustrate and examine the effectiveness of the proposed algorithm. The performance of the algorithm is evaluated in terms of following important parameters:

Mean square error in the pass band (E_p);
Stop band error (E_s);
stop-band first lobe attenuation (A_L);
stop-band edge attenuation(A_s) = $-20\log_{10}(H_0(\omega_s))$;
maximumreconstruction error (ϵ)in dB = $\max_{\omega}|10\log|T(e^{j\omega})||$

A MATLAB program has been written which implements the design technique described above.

4.1 Design example

***Example**:*For filter length $(N+1) = 48$, $\omega_s=0.6\pi$, $\omega_p=0.4\pi$,$\alpha_1=0.95$, $\alpha_2=0.04$,$\alpha_3 = 1$ and $\alpha_4 = 10^{-4}$, the following 24-filter coefficients for the FIR low-pass prototype filter are obtained:

$h_0(0)$=0.000208516673605 $h_0(1)$ =-0.000277085005004
$h_0(2)$ = -0.000485925710620 $h_0(3)$ =0.000939830506683
$h_0(4)$ =0.000828146074712 $h_0(5)$ = -0.002254669530186
$h_0(6)$=-0.001114175660144 $h_0(7)$=0.004518779506026
$h_0(8)$=0.001102671159255 $h_0(9)$=-0.008074502198911
$h_0(10)$=-0.000416487855403 $h_0(11)$=0.013299020942940
$h_0(12)$=-0.001522982128226 $h_0(13)$=-0.020679775981083
$h_0(14)$=0.005591974419610 $h_0(15)$=0.030998856797507
$h_0(16)$=-0.013310060758246$h_0(17)$=-0.045983099224892
$h_0(18)$= 0.027954420876814 $h_0(19)$=0.070646028734991
$h_0(20)$=-0.059744465465400 $h_0(21)$=-0.126933649477169
$h_0(22)$= 0.172673420719003 $h_0(23)$=0.593622292543939

The corresponding normalized magnitude plots of analysis filters$H_0(z)$and $H_1(z)$ are displayed in Fig.2a. Figures 2band 2c, respectively, depict theamplitude of distortion function and attenuation characteristics of low-pass filter $H_0(z)$. The reconstruction error (in dB) of QMF bankis plotted in Fig. 2d.The corresponding important parameters are E_p= 3.144×10^{-10},E_s= 2.96×10^{-8},A_s= 55.06dB,A_L= 64.09 dB, and PRE(ϵ) in dB = 0.0092dB.

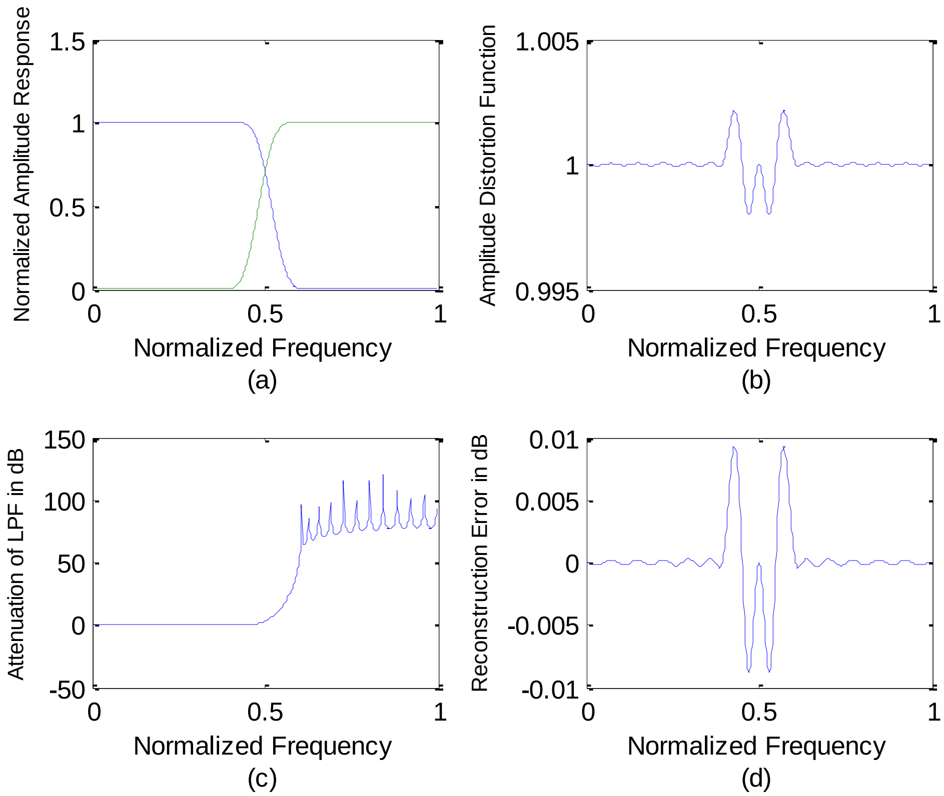


Fig.2 (a) Amplitude response of analysis filters for filter length = 48, (**b**) Amplitude of distortion function, (**c**) Attenuation characteristics of low-pass analysis filter,(**d**) Reconstruction error in dB.

From the above results, it is clearly indicate that the performance of the proposed method significantly improvedwhen compared than to earlier known techniques in terms of PRE, $E_p$and$A_s$with similar design specifications.

5 Conclusion

A new iterative method for the design of multirate filter banks has been developed by formulating the perfect reconstruction condition in the frequency domain. The quadratic objective function is minimized without any matrix inversion which generally affects the effectiveness of some methods. Design example showsthat the proposed technique is very effective in designing the quadrature mirror filters.The peak reconstruction error is minimumby the proposed method that makes it suitable for real time applications. Further, it is possible to extend this approach for the design of QMF banks with more than two bands.

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