SOME INFINITE AND FINITE SUMMATION FORMULAE INVOLVING *I*-FUNCTION

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ABSTRACT:

In this paper we shall obtain some infinite and finite summation formulae involving I-function. A large number of finite and infinite series for I-function can be summed up by using the well-known summation theorems for ordinary hypergeometric series in the main results. The method used in deriving these series is also simple and direct and does not impose heavy restrictions on the parameters. These summation formulae are of interest in themselves and yield, in turn, a large number of other results as their particular cases. We present one particular case of our main summation formulas, which are also novel and interesting on their own, as an illustration.

Keywords:

[1] INTRODUCTION

In 1997, Rathie has defined the *I*-function [3] by the following Mellin-Barnes type contour integral:

$$I_{R_l,T_l:Q}^{B,V}\left[u\left|\begin{matrix}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l}\\(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l}\end{matrix}\right.\right] = \frac{1}{(2\pi i)}\int_L \varphi(\xi)u^\xi d\xi ,$$

(1.1)

where

$$\varphi(\xi) = \frac{\prod_{k=1}^B \Gamma(\mu_k-\phi_k\xi)\prod_{k=1}^V \Gamma(1-\lambda_k+\psi_k\xi)}{\sum_{l=1}^Q\{\prod_{k=B+1}^{T_l}\Gamma(1-\mu_{kl}-\phi_{kl}\xi)\prod_{k=V+1}^{R_l}\Gamma(\lambda_{kl}-\psi_{kl}\xi)\}} .$$

(1.2)

Also

(i) $u \neq 0$;

(ii) $i= \sqrt{-1}$;

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(iii) B, V, $R_l(l = 1, \dots, Q)$ and $T_l(l = 1, \dots, Q)$ are integers satisfying $0 \leq V \leq R_l$, $1 \leq B \leq T_l(l = 1, \dots, Q)$, Q is finite.

(iv) L is a suitable contour in the complex plane.

(v) A product that is empty is taken to represent unity.

(vi) ψ_k, $(k = 1, \dots, R_l)$; ϕ_k $(k = 1, \dots, T_l)$; ψ_{kl} $(k = 1, \dots, R_l)$ and ϕ_{kl}, $(k = 1, \dots, T_l)$ are positive numbers.

(vii) ψ_k, $k = 1, \dots, R_l$ and ϕ_k, $k = 1, \dots, T_l$ are complex numbers such that no singularity of $\Gamma(\mu_k - \phi_k\xi)$, $k = 1, \dots, B$ coincides with any singularity of $\Gamma(1 - \lambda_k + \psi_k\xi)$, $k = 1, \dots, V$. In general these singularities are not poles.

The contours of L of integration comes in three distinct shapes.

(a) In order for all the singularities of $\Gamma(\mu_k - \phi_k\xi)$, $k = 1, \dots, B$, to lie to the right of L, and all the singularities of $\Gamma(1 - \lambda_k + \psi_k\xi)$, $k = 1, \dots, V$, to lie to the left of L, L travels from $\sigma - i\infty$ to $\sigma + i\infty$, (σ real).

(b) L is a clockwise loop that starts and ends at $+\infty$, encompassing all of the singularities of $\Gamma(\mu_k - \phi_k\xi)$, $k = 1, \dots, B$, once; however, it does not include any of the singularities of $\Gamma(1 - \lambda_k + \psi_k\xi)$, $k = 1, \dots, V$.

(c) L is a loop that encircles all of the singularities of $\Gamma(1 - \lambda_k + \psi_k\xi)$, $k = 1, \dots, V$, and starts and ends at $-\infty$., once in the clock-wise direction, but none of the singularities of $\Gamma(\mu_k - \phi_k\xi)$, $k = 1, \dots, B$.

[2] INFINITE SUMMATION FORMULAE

FIRST FORMULA

$$\sum_{\omega=0}^{\infty} \frac{(-d)_\omega}{\omega!} I_{R_l+3,T_l+4:Q}^{B,V+3} \left[u \left| \begin{matrix} (-\epsilon - \omega + 1, y), (-\zeta - \omega + 1, e), (-\epsilon - \zeta + \tau + d, y + e - p), \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon + 1, y), (-\zeta + 1, e), \\ (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \end{matrix} \right. \right.$$
$$\left. \left. \begin{matrix} (-\tau - \omega + 1, p), (-\epsilon - \zeta + \tau + d - \omega, y + e - p) \end{matrix} \right] \right]$$

$$= I_{R_l+3,T_l+4:Q}^{B+3,V} \left[u \left| \begin{matrix} (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l}, (-\epsilon + \tau, y - p), (-\zeta + \tau, e - p), \\ (-\epsilon + \tau + d, y - p), (-\zeta + \tau + d, e - p), (-\epsilon - \zeta + \tau, y + e - p), \\ (-\epsilon - \zeta + \tau + d, y + e - p) \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\tau - d + 1, p) \end{matrix} \right. \right],$$

(2.1)

where $e - p \geq 0, y - p \geq 0$ and $(y, e, p) \geq 0$ (not all zero simultaneously).

SECOND FORMULA

$$\sum_{\omega=0}^{\infty} \frac{1}{\omega!} I_{R_l+3,T_l+5:Q}^{B,V+3} \left[u \left| \begin{matrix} (-\epsilon - \omega + 1, y), (-\zeta - \omega + 1, e), (-\tau - \omega + 1, p), \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon + 1, y), (-\zeta + 1, e), (-\tau + 1, p), \\ (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \end{matrix} \right. \right.$$
$$\left. \left. \begin{matrix} (-\tau - \omega + 1, p), (-\epsilon - \zeta + \tau + d - \omega, y + e - p) \end{matrix} \right] \right]$$

$$= I_{R_l+2,T_l+4:Q}^{B,V+2} \left[u \left| \begin{matrix} \left(-\frac{\epsilon}{2}, \frac{y}{2}\right), \left(-\frac{\epsilon}{2} + \zeta + \tau, \frac{y}{2} - e - p\right), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon, y), \left(-\frac{\epsilon}{2} + \zeta, \frac{y}{2} - e\right), \left(-\frac{\epsilon}{2} + \tau, \frac{y}{2} - p\right) \\ (-\epsilon + \zeta + \tau, y - e - p) \end{matrix} \right. \right],$$

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$$(2.2)$$

where $\quad y - 2e \geq 0, y - 2p \geq 0, y - 2e - 2p \geq 0 \ and \ (y, e, p) \geq 0 \quad$ (not all zero simultaneously).

THIRD FORMULA

$$
\sum_{\omega=0}^{\infty} \frac{1}{\omega!} I_{R_l+4,T_l+5:Q}^{B,V+4} \left[u \left| \begin{array}{l} (-\epsilon - \omega + 1, y), (-\zeta - \omega + 1, e), (-\tau - \omega + 1, p), (-2\tau + 1, 2p), \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon + 1, y), (-\zeta + 1, e), (-\tau + 1, p), \\ (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \end{array} \right. \right.
$$
$$
\left. \left. \begin{array}{l} \left(-\frac{\epsilon}{2} - \frac{\zeta}{2} - \omega + \frac{1}{2}, \frac{y}{2} + \frac{e}{2}\right), (-2\tau - \omega + 1, 2p) \end{array} \right] \right.
$$
$$
= \sqrt{\pi}\, I_{R_l+3,T_l+3:Q}^{B+1,V+1} \left[u \left| \begin{array}{l} \left(-\tau + \frac{1}{2}, p; 1\right), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l}, \left(-\frac{\epsilon}{2} + \tau + \frac{1}{2}, \frac{y}{2} - p\right) \\ \left(-\frac{\epsilon}{2} - \frac{\zeta}{2} + \tau + \frac{1}{2}, \frac{y}{2} + \frac{e}{2} - p\right), (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, \\ \left(-\frac{\zeta}{2} + \tau + \frac{1}{2}, \frac{e}{2} - p\right) \\ \left(-\frac{\epsilon}{2} + \frac{1}{2}, \frac{y}{2}\right), \left(-\frac{\zeta}{2} + \frac{1}{2}, \frac{e}{2}\right) \end{array} \right. \right],
$$

$$(2.3)$$

where $\quad y - 2p \geq 0, e - 2p \geq 0 \ and \ (y, e, p) \geq 0 \quad$ (not all zero simultaneously) and $Re\left(\frac{1}{2} - \frac{\epsilon}{2} - \frac{\zeta}{2} + \tau\right) > 0.$

FOURTH FORMULA

$$
\sum_{\omega=0}^{\infty} \frac{1}{\omega!} I_{R_l+4,T_l+5:Q}^{B+1,V+3} \left[u \left| \begin{array}{l} (-\epsilon - \omega + 1, y), (-\tau - \omega + 1, e), (-2\tau + k, 2e - p), \\ (-\epsilon + \omega + 1, y), (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon + 1, y), (-\tau + 1, p), \\ (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l}, (-\epsilon + 1, y) \end{array} \right. \right.
$$
$$
\left. \left. \begin{array}{l} (-k - \omega + 1, p), (-2\tau + k - \omega, 2e - p) \end{array} \right] \right.
$$
$$
= \pi\, 2^{1-2\tau}\, I_{R_l+3,T_l+2:Q}^{B,V+1} \left[2^{-2e} u \left| \begin{array}{l} (-2\tau + k, 2e - p,), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l}, \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, \end{array} \right. \right.
$$
$$
\left. \left. \begin{array}{l} \left(-\frac{\epsilon}{2} + \frac{k}{2} + \frac{1}{2}, \frac{y}{2} - \frac{p}{2}\right), \left(-\frac{\epsilon}{2} + \tau + \frac{k}{2} + 1, \frac{y}{2} - e + \frac{p}{2}\right) \\ \left(-\frac{\epsilon}{2} - \frac{k}{2} + 1, \frac{y}{2} + \frac{p}{2}\right), \left(-\frac{\epsilon}{2} - \tau + \frac{k}{2} + \frac{1}{2}, \frac{y}{2} + e - \frac{p}{2}\right) \end{array} \right] \right.,
$$

$$(2.4)$$

Where $2e - p \geq 0, y - 2e \geq 0 \ , y - p \geq 0 \ and \ (y, e, p) \geq 0$ (not all zero simultaneously) and $Re(k - \epsilon - \zeta - \tau) > 0.$

FIFTH FORMULA

$$
\sum_{\omega=0}^{\infty} \frac{(-d)_\omega}{\omega!} I_{R_l+9,T_l+12:Q}^{B,V+9} \left[u \left| \begin{array}{l} (-\epsilon - \omega + 1, y), \left(-\frac{\epsilon}{2} - \omega, \frac{y}{2}\right), (-\zeta - \omega + 1, e), \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1,T_l}, (-\epsilon + 1, y), \left(-\frac{\epsilon}{2} + 1, \frac{y}{2}\right), (-\zeta + 1, e), \end{array} \right. \right.
$$

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$$(-\tau - \omega + 1, p), (-\rho - \omega + 1, \theta), (-k - \omega + 1, \phi), \left(-\frac{\epsilon}{2} + 1, \frac{y}{2}\right), (-\epsilon + k, y - \phi),$$

$$(-\tau + 1, p), (-\rho + 1, \theta), (-k + 1, \sigma), \left(-\frac{\epsilon}{2} - \omega + 1, \frac{y}{2}\right), (-\epsilon + \zeta - \omega, y - e),$$

$$(-\epsilon - d, y), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l},$$

$$\left. (-\epsilon + \tau - \omega, y - p), (-\epsilon + \rho - \omega, y - \theta), (-\epsilon + k - \omega, y - \sigma), (-\epsilon - d - \omega, y) \right]$$

$$= I_{R_l+5, T_l+8:Q}^{B, V+5} \left[u \left| \begin{matrix} (-\epsilon - d, y), (-\epsilon + \zeta + \tau - d, y - e - \theta), (-\epsilon + \tau + \rho - d, y - e - \theta), \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1, T_l}, (-\epsilon, y), (-\epsilon + \zeta + \tau, y - e - p), \end{matrix} \right. \right.$$
$$(-\epsilon + \tau + \rho - d, y - p - \theta), (-\epsilon + \zeta + \tau + \rho, y - e - p - \theta)$$
$$(-\epsilon + \zeta + \rho - d, y - e - \theta), (-\epsilon + \zeta + \rho - d, y - p - \theta), (-\epsilon + \zeta - d, y - e),$$
$$\left. (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \right.$$
$$\left. (-\epsilon + \tau - d, y - p), (-\epsilon + \rho - d, y - \theta), (-\epsilon + \zeta + \tau + \rho - d, y - e - p - \theta) \right],$$

$$(2.5)$$

where $y - \sigma \geq 0, y - e \geq 0, y - p \geq 0, y - \theta \geq 0, y - e - p \geq 0, y - e - \theta \geq 0, y - p - \theta \geq 0, y - e - p \geq 0, y - e - p - \theta \geq 0$, $(y, e, p, \theta, \sigma \geq 0)$ (not all zero simultaneously), $1 + 2\epsilon = \zeta + \tau + \rho + k - d$ and $d = 0,1,2, \dots$.

In all the above formulae (2.1) through (2.5), the various *I*-function satisfy the conditions of convergence corresponding appropriately to the conditions of *I* – function.

PROOFS:

Using definition (1.1), we substitute the Mellin-Barnes integral for the I-function on the left-hand side of, say, (2.1), and then we switch the sequence of integration and summation (which is easily justified when the integral and the involved series are absolutely convergent). We find that

$$\Delta = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{\varphi(\xi)}{\Gamma(\tau + p\xi)} \left[\sum_{\omega=0}^{\infty} \frac{(\epsilon + y\xi)_\omega (\zeta + e\xi)_\omega (-d)_\omega}{\omega! \ (\tau + p\xi)_\omega (1 + \epsilon + \zeta - \tau - d + (y + e - p)\xi)_\omega} \right] u^\xi d\xi$$

This gives us the right hand side of (2.1) when we use the Salzschütz's formula [1] and use (1.1) to understand the answer that is thus obtained.
The results ([15], [16]) can be used to create proof of (2.2) through (2.5) on lines similar to those above.

[3] FINITE SUMMATION FORMULAE

FIRST FORULA

$$\sum_{\omega=0}^{\infty} \frac{1}{\omega!} I_{R_l+2, T_l+3:Q}^{B, V+2} \left[u \left| \begin{matrix} (-\epsilon - \omega + 1, y), (-\zeta - \omega + 1, e), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1, T_l}, (-\epsilon + 1, y), (-\zeta + 1, e), (-\epsilon - \zeta - \omega, y + e) \end{matrix} \right. \right]$$

$$= \frac{1}{d!} I_{R_l+2, T_l+3:Q}^{B, V+2} \left[u \left| \begin{matrix} (-\epsilon - d, y), (-\zeta - d, e), (\lambda_k, \psi_k)_{1,V}, (\lambda_{kl}, \psi_{kl})_{V+1,R_l} \\ (\zeta_k, \phi_k)_{1,B}, (\mu_{kl}, \phi_{kl})_{B+1, T_l}, (-\epsilon, y), (-\zeta, e), (-\epsilon - \zeta - d, y + e) \end{matrix} \right. \right],$$

$$(3.1)$$

where $y, e \geq 0$ (not all zero simultaneously).

SECOND FORMULA

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$$\sum_{\omega=0}^{\infty}\frac{1}{\omega!}I_{R_l+4,T_l+4:Q}^{B,V+3}\left[u\left|\begin{array}{l}(-\epsilon-\omega+1,y),(-\zeta-\omega+1,e),(-\tau-\omega+1,p),\\(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-\epsilon+1,y),(-\zeta+1,e),\end{array}\right.\right.$$
$$\left.\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(\rho+\omega,\theta)\\(-\tau+1,p),(-\epsilon-\zeta-\tau+\rho-\omega+1,y+e+p+\theta)\end{array}\right]$$
$$=\frac{1}{d!}I_{R_l+4,T_l+4:Q}^{B,V+3}\left[u\left|\begin{array}{l}(-\epsilon-d,y),(-\zeta-d,e),(-\tau-d,p),\\(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-\epsilon,y),(-\zeta,e),\end{array}\right.\right.$$
$$\left.\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(\rho+d,\theta)\\(-\tau,p),(-\epsilon-\zeta-\tau+\rho-d+1,y+e+p+\theta)\end{array}\right],$$

(3.2)

where $y,e,p,\theta\geq 0$ (not all zero simultaneously) and $\zeta\tau+\tau\epsilon+\epsilon\zeta=(\rho-1)(\epsilon+\zeta+\tau-\rho-1)$.

THIRD FORMULA

$$\sum_{\omega=0}^{\infty}\frac{\{\Gamma(1+v+\omega)\}^{-1}}{\omega!}I_{R_l+3,T_l+3:Q}^{B+2,V+1}\left[u\left|\begin{array}{l}(-s-1,a)(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,b),(v-s-1,a)\\(v-s+\omega,b),(v-s+\omega-1,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s,a)\end{array}\right.\right]$$
$$-I_{R_l+3,T_l+3:Q}^{B+2,V+1}\left[u\left|\begin{array}{l}(-t-1,b)(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s-1,b),(v-s,a)\\(v-s+\omega-1,b),(v-s+\omega,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s,a)\end{array}\right.\right]$$
$$=\frac{\{\Gamma(1+v+d)\}^{-1}}{d!}I_{R_l+3,T_l+3:Q}^{B+2,V+1}\left[u\left|\begin{array}{l}(-s,a)(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,b),(v-s,a)\\(v-s+d,b),(v-s+d,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-t,a)\end{array}\right.\right]$$
$$-I_{R_l+3,T_l+3:Q}^{B+2,V+1}\left[u\left|\begin{array}{l}(-t,b)(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,b),(v-s,a)\\(v-s+d,b),(v-s+d,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s+1,a)\end{array}\right.\right],$$

(3.3)

where $a,b\geq 0$ (not both zero simultaneously).

FOURTH FORMULA

$$\sum_{\omega=0}^{\infty}\frac{1}{\omega!}\left\{I_{R_l+3,T_l+2:Q}^{B+2,V}\left[u\left|\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,k),(v-s-1,z),(v+\omega+1,z)\\(v-t+\omega,k),(v-s+\omega-1,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l}\end{array}\right.\right]\right.$$
$$\left.-(1+t)I_{R_l+3,T_l+2:Q}^{B+2,V}\left[u\left|\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t-1,k),(v-s,z),(v+\omega+1,z)\\(v-t+\omega,k),(v-s+d,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l}\end{array}\right.\right]\right\}$$
$$=\frac{(s-t)}{d!}I_{R_l+3,T_l+2:Q}^{B+2,V}\left[u\left|\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,k),(v-s,z),(v+\omega+1,z)\\(v-t+d,k),(v-s+\omega,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l}\end{array}\right.\right],$$

(3.4)

where $z\geq 0$.

FIFTH FORMULA

$$\sum_{\omega=0}^{\infty}\frac{1}{\omega!}\left\{\frac{(v-t)_{\omega}}{(v+1)_{\omega}}I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[u\left|\begin{array}{l}(-s-1,a),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s-1,a)\\(v-s+\omega-1,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s,a)\end{array}\right.\right]\right.$$
$$\left.-\frac{(1+t)(v-t-1)_{\omega}}{(v+1)_{\omega}}I_{R_l+1,T_l+1:Q}^{B+1,V}\left[u\left|\begin{array}{l}(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,a)\\(v-s+\omega,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l}\end{array}\right.\right]\right\}$$
$$=\frac{(v-t)_{\omega}}{d!(v+1)_{\omega}}I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[u\left|\begin{array}{l}(t-s,a),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,a)\\(v-s+d,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(t-s+1,a)\end{array}\right.\right],$$

(3.5)

where $a\geq 0$.

Now, we give very elegant results involving the product of two I – functions:

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SIXTH FORMULA

$$\sum_{\omega=0}^{\infty} \frac{\{\Gamma(1+v+\omega)\}^{-1}}{\omega!} \left\{ I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[x \left| \begin{array}{l} (-s-1,a),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s-1,a) \\ (v-s+\omega-1,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s,a) \end{array} \right. \right] \right.$$

$$\times I_{R_l+1,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,b) \\ (v-t+\omega,b),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$- I_{R_l+1,T_l+1:Q}^{B+1,V}\left[x \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,a) \\ (v-s+\omega,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$\left. \times I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[y \left| \begin{array}{l} (-t-1,b),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t-1,b) \\ (v-t+\omega-1,b),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-t,b) \end{array} \right. \right] \right\}$$

$$= \frac{\{\Gamma(1+v+d)\}^{-1}}{d!} \left\{ I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[x \left| \begin{array}{l} (-s,a),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,a) \\ (v-s+d,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-s+1,a) \end{array} \right. \right] \right.$$

$$\times I_{R_l+1,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,b) \\ (v-t+d,b),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$- I_{R_l+1,T_l+1:Q}^{B+1,V}\left[x \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,a) \\ (v-s+d,a),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$\left. \times I_{R_l+2,T_l+2:Q}^{B+1,V+1}\left[y \left| \begin{array}{l} (-t,b),(\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,b) \\ (v-t+d,b),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l},(-t+1,b) \end{array} \right. \right] \right\},$$

$$(3.6)$$

where $a,b \geq 0$ (not all zero simultaneously).

SEVENTH FORMULA

$$\sum_{\omega=0}^{\infty} \frac{1}{\omega!} \left\{ (1+s) I_{R_l+2,T_l+1:Q}^{B+1,V}\left[x \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s-1,z),(v+\omega+1,z) \\ (v-s+\omega-1,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right] \right.$$

$$\times I_{R_l+1,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,z) \\ (v-t+\omega,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$- (1+t) I_{R_l+2,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,z),(v+\omega+1,z) \\ (v-s+\omega,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$\left. \times I_{R_l+1,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t-1,z) \\ (v-t+\omega-1,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right] \right\}$$

$$= \frac{(s-t)}{d!} I_{R_l+2,T_l+1:Q}^{B+1,V}\left[x \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-s,z),(v+d+1,z) \\ (v-s+d,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$\times I_{R_l+1,T_l+1:Q}^{B+1,V}\left[y \left| \begin{array}{l} (\lambda_k,\psi_k)_{1,V},(\lambda_{kl},\psi_{kl})_{V+1,R_l},(v-t,z) \\ (v-t+d,z),(\zeta_k,\phi_k)_{1,B},(\mu_{kl},\phi_{kl})_{B+1,T_l} \end{array} \right. \right]$$

$$(3.7)$$

where $z \geq 0$.

In all the above formulae (3.1) through (3.7), the various I–functions satisfy the conditions of convergence corresponding appropriately to the conditions of I-function.

PROOFS:

Proof for (3.1) to (3.7) may be established along the same lines as previously stated by utilizing well-known findings ([16], p.84, eqs. (2.6.1.9), (2.6.1.10)).

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where just the first n terms of the F series are to be included in the expansion, as indicated by the suffix n on the left.
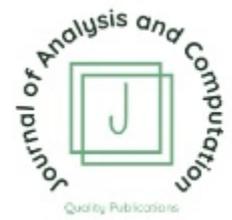
In (3.3) through (3.6), putting, and $R = 1$ we arrive at some finite summation formulae obtained earlier by R. C. Manglik [19].

[4] SUMMARY

In this paper, we have presented certain infinite and finite summation formulae involving I-function. Our findings offer intriguing consolidation and expansions of several established and novel findings. A large number of finite and infinite series for *I*-function can be summed up by using the well-known summation theorems for ordinary hypergeometric series in the main results. The method used in deriving these series is also simple and direct and does not impose heavy restrictions on the parameters.

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