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MODELLING OF EFFECTIVE THERMAL CONDUCTIVITY OF HIGH POROSITY PERIODIC MATERIALS

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ABSTRACT:

A theoretical model for predicting effective thermal conductivity (ETC) of highly porous two-phase systems has been developed. The arrangement has been divided into unit cells .For determining the effective thermal conductivity ,the unit cell can be divided into two or three layers in series .The conductivity of each layer, in turn, is derived by applying the parallel law of thermal resistances. By combining these layers which are in series the effective thermal conductivity of unit cell is derived. The theoretical expressions so obtained values of effective thermal conductivity that are quit close to the experimental results.

Keywords: *Effective thermal conductivity, Unit cells, Porous media, Parallel and Series resistors model.*

[1] INTRODUCTION:

The knowledge of the effective thermal conductivity of high porous metal foam is becoming increasingly important in the technological developments and in many engineering applications. Kaviany (1995) has provided an extensive review of the available literature on the subject along with a number of correlations and their range of applicability. Under simplified one dimensional conduction condition, two extremes can be considered, one in which the thermal resistance offered by the solid and fluid phases are in series and the other in which they are in parallel. The upper bound given by equation (i)

$$\lambda_e = (1-\phi)\lambda_s + \phi\lambda_f \qquad \textbf{(1)}$$

has been successfully used in the past for packed bed Studies. Where the solid and fluid phases have similar conductivities. However, the error in the prediction of ke using equation (i) can be considerable as the difference in the conductivities of the phases increases. There have been several studies that attempt to predict the thermal conductivities of packed beds by invoking the structure of the medium, apart from its porosity. Some of these two and one dimensional studies are reviewed below. Many of them are related to packed beds of spheres and granular materials.

Nozad et al. (1985) solved the two dimensional heat conduction equation in a spatially periodic two phase systems of touching spheres. Under the assumption of local thermal equilibrium (Carbonell and Whitakar, 1984) they derived a set of closure equations for the spatial deviation component of the volume –averaged temperature field in the constitutive phases. These closure equation where analytically and numerically to obtain the effective thermal conductivity. Their numerical results correlated well with the experimental data for a touching parameter c/a = 0.02 (c/a is the ratio of the touching length scale to the length scale of particle size) based on a similar two-dimensional study, Sahroui and kaviany (1993) found a value of c/a = 0.002 to be more appropriate. Hsu et al. (1995) demonstrated that a one dimensional conduction model based on in line touching cubes (c/a = 0.13) was sufficient. It showed good agreement with the experimental data of a packed sphere bed.

Hsu et al. (1994) based their study on an earlier work of Zehner and Schlunder (1970). They proposed two models-the area contact model for packed beds of spheres and the phase symmetry model for sponge like materials (fibrous media). They showed that for Packed beds of spheres, their area contact model was able to predict the thermal conductivity better than the Zehner- Schlunder model because it took into account the finite contact area between adjacent particles. They also developed a phase symmetric model for sponger like porous media (e.g.metal foams).Bauer (1993) generalized Maxwell's classical theoretical result to pores of any shape and concentration. He further discussed cases in which other phenomena-like radiation can be included into the analysis.

Tien and Vafai (1979) derived statistical bounds for the thermal conductivity of micro sphere and fibrous insulations based on cell geometies. Their model for fibrous Insulations was used by Hunt and Tien (1980) to study forced convection in the metal foams. Calmidi and Mahajan et al. (1) also developed a model to predict effective thermal conductivity. Our survey indicates a lock of experimentally validated studies that attempt to predict the Effective thermal conductivity of metal foams, although they have been the topic of investigation for quite a while. Further, in order to explore their use as high performance heat sinks, an accurate estimate to the effective thermal conductivity is necessary. In the study, we report our measurement of effective thermal conductivity of metal foam made of aluminum.

Experiments were performed with air and water the fluid phase separately. An empirical relation that correlates the experimental data to an accuracy of 97.5 percent is developed. Taking the foam structure to be square and hexagonal, an analytical model for the effective thermal conductivity is derived and validated with the experimental data.

Table-1 Metal foam properties and thermal conductivity

No.	Porosity	Pore density (pores/inch)	Conductivity (air +foam) (w/m-k)	Conductivity (water +foam) (w/m-k)
1.	0.971	5	2.70	3.70
2.	0.946	5	4.60	5.40
3.	0.905	5	6.70	7.65
4.	0.949	10	4.00	4.95
5.	0.909	10	6.70	7.60
6.	0.978	20	2.20	3.05
7.	0.949	20	3.90	4.80
8.	0.906	20	6.90	7.65
9.	0.972	40	2.50	3.30
10.	0.952	40	3.90	4.75
11.	0.937	40	4.50	5.35

2. THEORITICAL FORMULATION:

We assume one dimensional heat conduction in order to derive an analytical expression for the conductivity. The direction of heat flow is as shown in the figure 2. note that constituent with the assumption of one dimensional conduction the side faces are adiabatic. Although this assumption may note be true locally, globally the heat transfer is needed one dimensional. For determining the effective thermal conductivity the unit cell in fig.2. can be divided in turn is derived separately by applying the parallel law of thermal resistances.

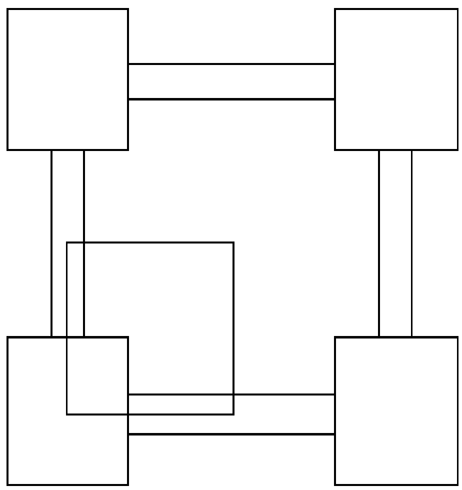

Fig. 1

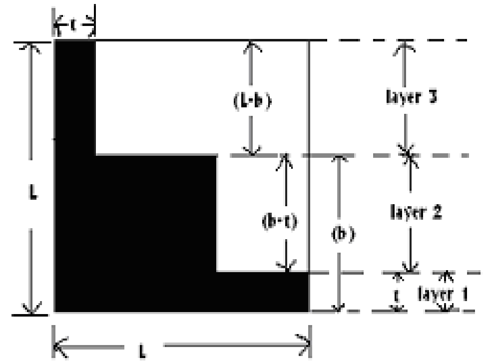

Fig.2

In layer 1st the solid and fluid phases are in parallel. Their respective volumes are given By

$$V_s = Ltw \qquad (2)$$

'w' in equation (ii) is the with of in the third direction (perpendicular to the plane of the paper).The conductivity of layer 1st can be written as

$$V_f = 0 \qquad (3)$$

$$\lambda_1 = \frac{Ltw}{Ltw} \lambda_{s_s} \qquad (3)$$

$$\lambda_1 = \lambda_2 \qquad (4)$$

Where $\lambda_s \rightarrow$ Conductivity of solid

$\lambda_{f\rightarrow}$ Conductivity of fluid

In a similar manner, for layer 2nd

The solid and fluid volume fraction are given by

$$V_s = (b-t)bw \qquad (5)$$

$$V_f = (b-t)(L-b)w \qquad (6)$$

The conductivity of layer 2nd can be written as

$$\lambda_{II} = \frac{(b-t)bw}{(b-t)Lw}\lambda_s + \frac{(b-t)(L-b)w}{(b-t)(L-w)w}\lambda_f \qquad (7)$$

For layer 3rd
The solid and fluid volume fraction are given by

$$V_s = t(L-b)w \qquad (8)$$

$$V_f = (L-b)(L-t)w \qquad (9)$$

The conductivity of layer 3rd can be written as

$$\lambda_{iii} = \frac{t(L-b)w}{L(l-b)w}\lambda_s + \frac{(l-b)(L-t)w}{L(L-b)w}\lambda_f \qquad (10)$$

In equation (10), an "area ratio" r is defined as

$$r = \frac{t}{b} \qquad (11)$$

By combining the three layers which are in series, the effective thermal conductivity of

$$\frac{L_1 + L_2 + L_3}{\lambda_{eff}} = \frac{L_1}{\lambda_I} + \frac{L_2}{\lambda_{II}} + \frac{L_3}{\lambda_{III}} \qquad (12)$$

Where λ_I, λ_{II} *and* λ_{III} are given by Eqs. (4), (7), and (10), respectively, and L_1, $L_{2,}$ *and* L_3 are the heights of the three layers in Fig. (2).
The fluid volume fraction ϕ is the ratio of the fluid volume to the volume of the unit

$$(\phi) = 1 - \frac{b^2}{L^2} - \frac{2t}{L^2}(L-b) \qquad (13)$$

The solid volume fraction can be written as

$$(1-\phi)=\frac{b^2}{L^2}+\frac{2t}{L}-\frac{2tb}{L^2} \qquad (14)$$

Putting the value of 't' from the Eqs.(11) to Eqs.(14)

$$(1-\phi)=\frac{b^2}{L^2}+\frac{2rb}{L}-\frac{2rb^2}{L^2}$$

$$(1-\phi)=(1-2r)\frac{b^2}{L^2}+\frac{2rb}{L}$$

$$(\frac{b}{L})^2(1-2r)+2r(\frac{b}{L})-(1-\phi)=0 \qquad (15)$$

Solution of Equation (15) can be written as

$$(\frac{b}{L})=\frac{-2r\pm\sqrt{(2r)^2-4(1-2r)(-(1-\phi))}}{2(1-2r)}$$

$$(\frac{b}{L})=\frac{-2r\pm\sqrt{4r^2+4(1-2r)(1-\phi)}}{2(1-2r)} \qquad (16)$$

b/L is plotted in Fig.5 for different values of r. Equation
(16) can be used in Eq. (12) to obtained the effective thermal conductivity of the unit cell in terms of the porosity "ϕ" and the area ratio "r". The final expression can be written as

$$\lambda_{eff}=[\frac{r(b/L)}{\lambda_I}+\frac{(1-r)(b/L)}{\lambda_{II}}+\frac{(1-(b/L))}{\lambda_{III}}]^{-1} \qquad (17)$$

where b/L is given by Eq. (16).

To assess the validity of this model, we first consider the experimentally measured values for metal foam and air given in column 4 of Table 1.They are plotted in Fig.5 as function of $1-\phi$, the solid fraction. Different symbols have been used for samples of different pore sizes. Clearly, an excellent fit between the experimental data and the predicted values is obtained for r = 0.09.

Fig. (9) shows the effective thermal conductivity of foamed materials $(\frac{\lambda_e}{\lambda_f})$ as a function of the porosity for different values of $\frac{\lambda_s}{\lambda_f}$. Also shown, along with experimental data, are curves for air $(\frac{\lambda_s}{\lambda_f} = 8226)$ and water $(\frac{\lambda_s}{\lambda_f} = 357)$. As expected, for $\lambda_s / \lambda_f = 1, \lambda_e = \lambda_f$ for all porosity values. As λ_s / λ_f increases, λ_e / λ_f also increases and all curves coverage at $\lambda_e / \lambda_f = 1$.

The effective thermal conductivity have been calculated using equation (13) & (16) for both aluminum/air and aluminum/water systems. The experimental results of effective thermal conductivity for the sample are shown with computed values. The theoretical results of effective thermal conductivity of our model are quite close to the experimental results of effective thermal conductivity for same samples. As was seen that the area ratio r = 0.09 results is excellent agreement with experimental results for both aluminum/air and aluminum/water systems. Due to the to dimensional nature of the assumed geometry, the ratio of the length scales of the fiber thickness to the intersection size is r1/2 = 0.3. A close examination of the metal foam structure indices that r is not unique and it infect varies. However r1/2 = 0.3 appears to be a representative value. Due to the one dimensional conduction analysis. It is expected that the value of r is slightly under- predicted since spreding effects are not included. Nevertheless, these effects are expected to be small due the high conductivity of aluminum. The experimental data for both air and water suggest that there is no symmetry effect of the pore density variation on the effective thermal conductivity. The implication is that the structure of the metal foam is more or less constant over the range of pore densities considered.

A comparison of the analytical (phase-symmetric) model of Hsu et al. (1994) with our experimental data indicates that their model over predicts the thermal conductivity for metal

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foams. Result of Zehner and Schlunder(1970) to be low fluid conductivity asymptote by invoking phase symmetry among the solid and fluid phases. It was found that in general, the data were close to the upper bound and orders of magnitude different from the lower bound. It is expected that the experimentally validated model for the thermal conductivity will be helpful in evaluation of metal foams as possible candidates as heat sinks in electronic cooling applications.

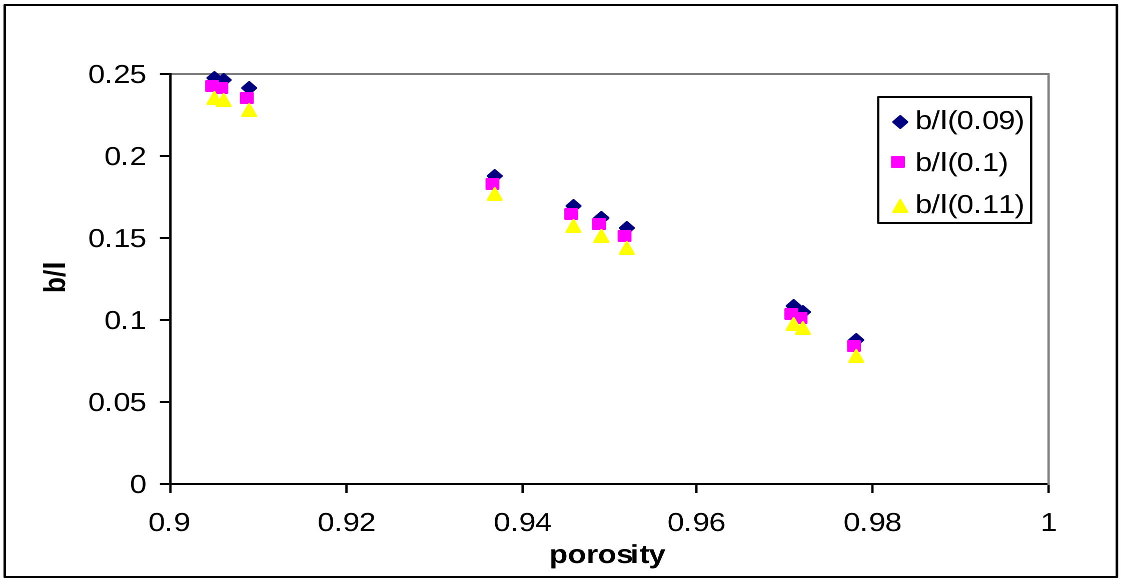

Fig.3 b/l (Eq. (16) as a function of ϕ

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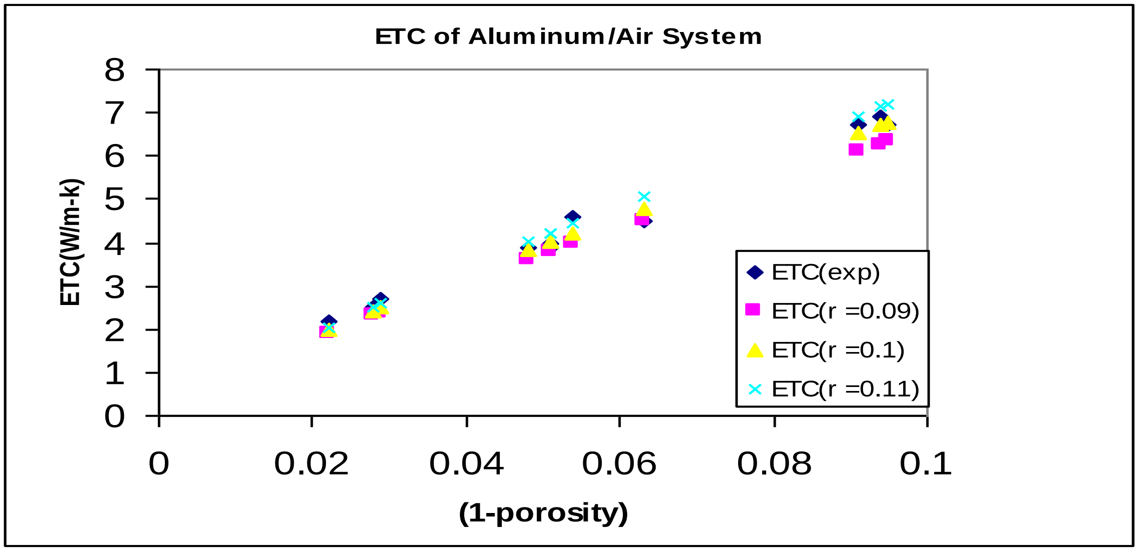

Fig.4 Comparison of experimental results with model (Eq. (17)) for

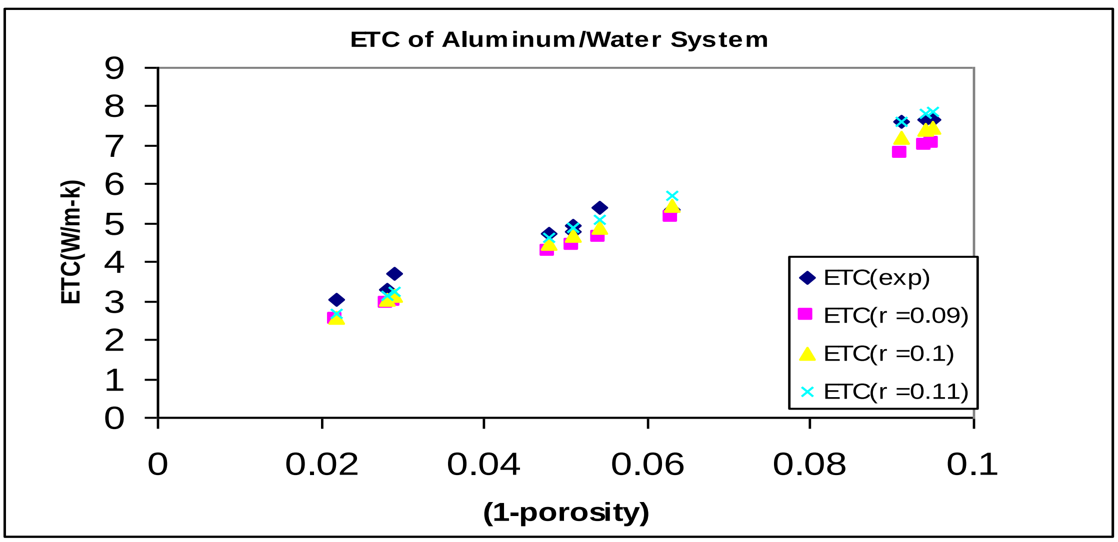

Fig.5 Comparison of experimental results with model (Eq. (17)) for

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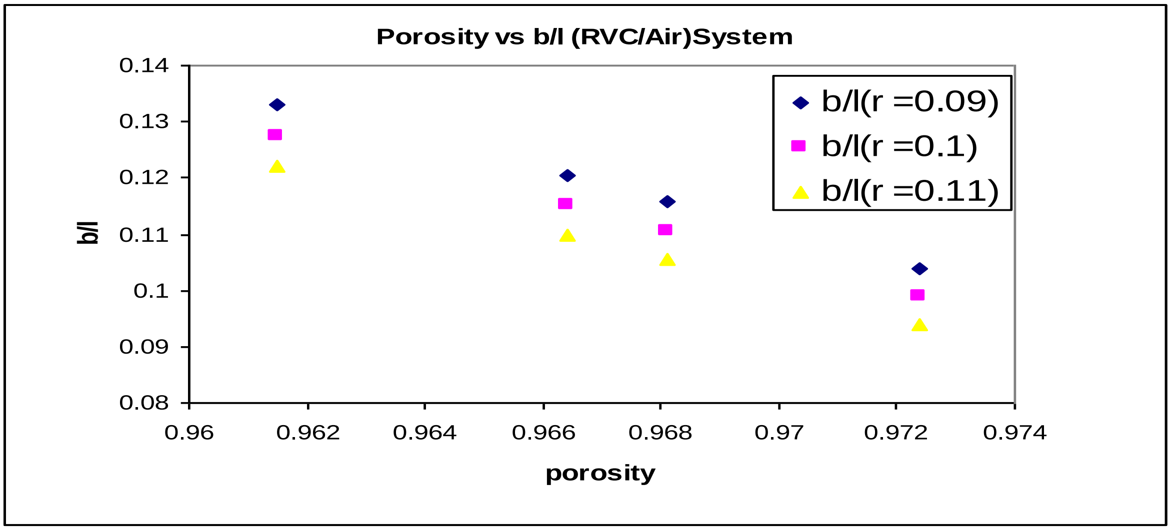

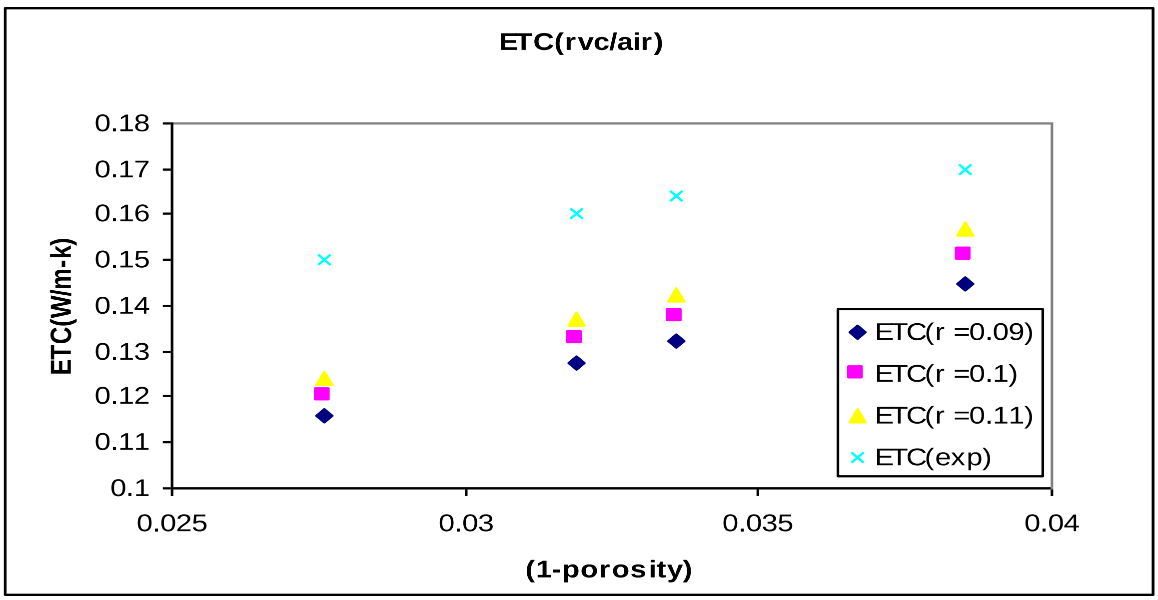

Fig.7 Comparison of experimental results with model (Eq. (17) for rvc/ air system $\lambda_f = (0.0265)W/m-k$

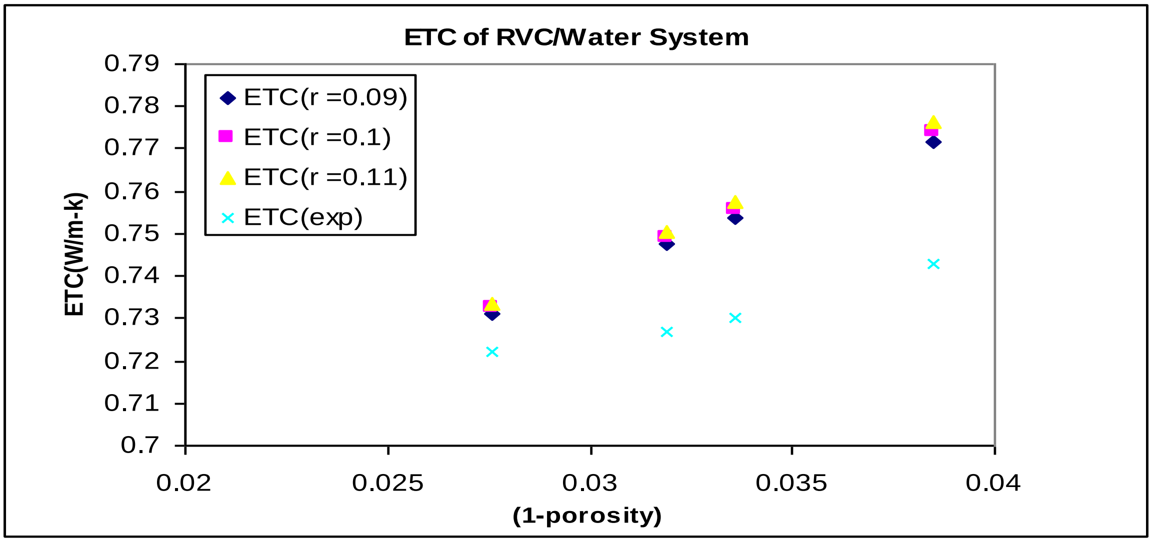

Fig.8. Comparison of experimental results with model (Eq.(17)) for water

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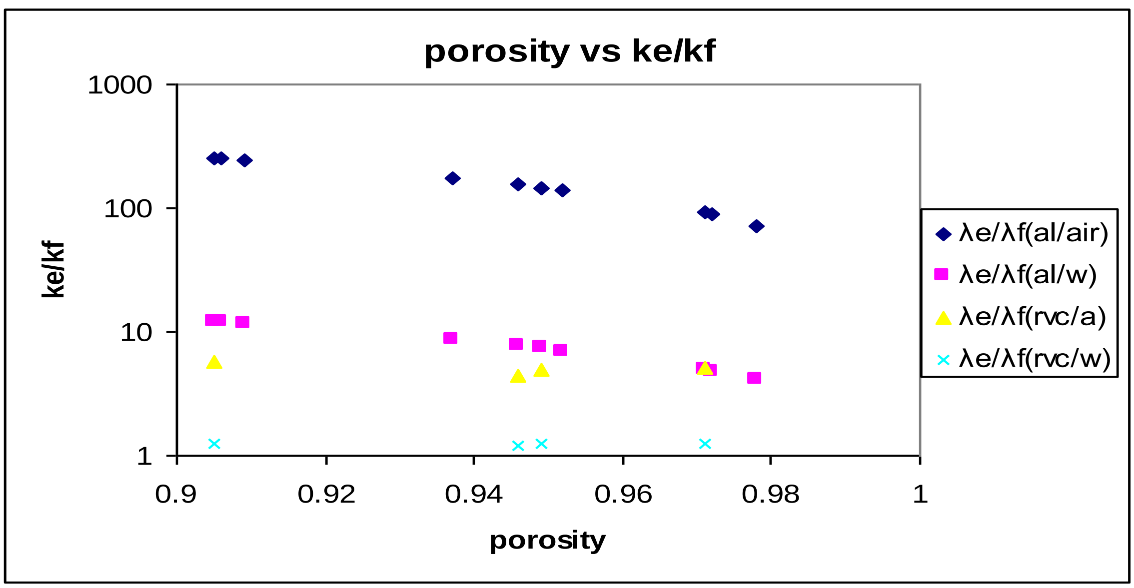

Fig.9 Effective thermal conductivity of foamed materials using equation (17) with r=0.09. Data for aluminum/air and aluminum/water is also shown, with

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